SOLUTIONS

PREALGEBRA

Contents

REAL NUMBER SET

*set = a collection of things (like a collection of marbles)

*set of numbers = a collection of numbers

TYPES OF NUMBERS SETS (CONSTITUTE REAL NUMBERS)
$\boldsymbol{natural\ numbers}$ $(or\ positive\ integers)$ $= \{1, 2, 3, 4, 5, \ldots\}$ $\boldsymbol{whole\ numbers} = \{0, 1, 2, 3, 4, 5, \ldots\}$ $\boldsymbol{negative\ integers} = \{-1, -2, -3, -4, -5, \ldots\}$ $\boldsymbol{integers} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ $\boldsymbol{rational\ numbers} = \left\{\frac{p}{q} : p, q\ integers, q \neq 0\right\}$ (numbers with repeating or terminating decimals) $\boldsymbol{irrational\ number}$ $are\ nonrepeating\ and\ nonterminating\ decimals$ (examples: $\pi = 3.14159\ldots$ and $\sqrt{2} = 1.4142\ldots$)

***Notice:** The whole numbers are the natural numbers with 0 included. **Circle the 0 in the whole numbers on the previous chart.**

*The whole numbers **contain** the natural numbers so we call the natural numbers a subset of the whole numbers.

*subset = a smaller collection of things from a bigger collection (like the set of blue marbles are a subset of all marbles)

***Notice:** The integers are the whole numbers with the negative integers included. **Circle the negative integers in the integers set in the chart above.**

***Notice:** The rational numbers are a quotient (or fraction) of integers where 0 cannot be in the bottom.

EXERCISE. (USE FIG.1 ON RIGHT)

1. **Place the point of your pencil inside the box that says 'Natural numbers'.**

2. **Notice that this box is inside the box that says 'Whole numbers', so your pencil tip is also inside 'Whole numbers'.**

3. **Translate this as "The Natural numbers are a subset of the Whole numbers."**

4. **Keep your pencil tip where it is. Is it inside the 'Negative Integers'? No, so the 'Natural numbers' are not contained within the 'Negative Integers'.**

Answer the following questions using this technique. Answer 'yes' or 'no'.

5. **Place your pencil tip in the 'Negative Integers' box.**
- Are the 'Negative Integers' in the 'Whole numbers'?

- Are the 'Negative Integers' in the 'Integers'?

- Are the 'Negative Integers' in the 'Rational numbers'?

6. **Place your pencil tip in the 'Whole numbers'.**
- Are the 'Whole numbers' in the 'Integers'?

- Are the 'Whole numbers' in the 'Rational numbers'?

- Are the 'Whole numbers' a subset of the 'Irrational numbers'?

- Are the 'Whole numbers' in the 'Natural numbers'? (Hint: Since the number 0, is not a natural number, not all whole numbers are natural numbers.)

7. **Place your pencil tip in the 'Integers' box.**
- Are the 'Whole numbers', 'Natural numbers', and 'Negative Integers', inside the 'Integers' box?

- Are the all 'Rational numbers' within the 'Integers' box?

- Are the 'Integers' a subset of the 'Irrational numbers'?

- Are the 'Integers' a subset of the 'Real numbers'?

* The goal of the previous exercise was to accustom you to think abstractly about sets of numbers. There are infinite real numbers so in order to understand the relationships that exist between the different types, it is best to study a diagram such as Fig. 1.

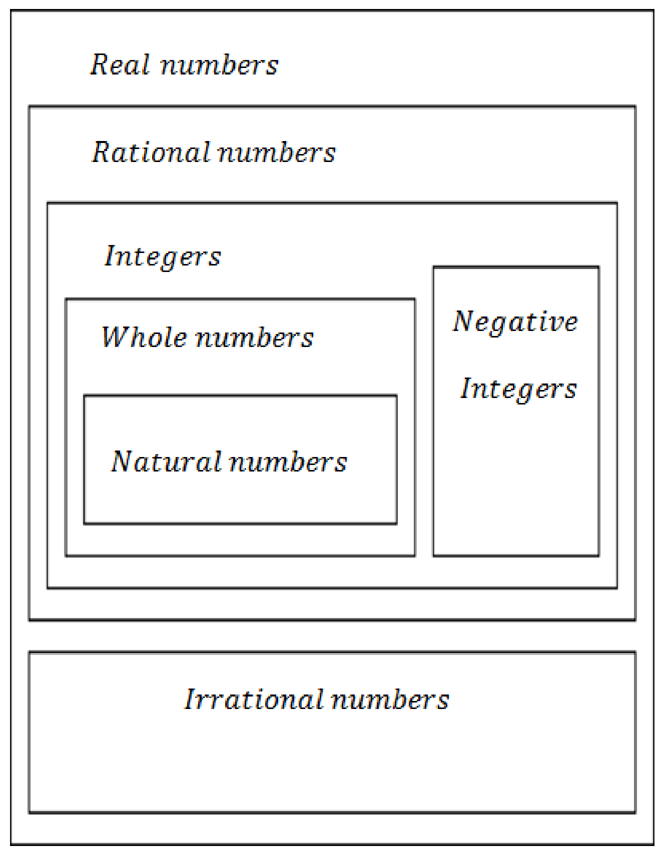


Fig.1

SOME FACTS FROM FIG.1

- The Real numbers contain all the number sets.

- The Rational numbers and Irrational numbers share no common number(s).

- All Natural numbers are in Whole numbers, Integers, Rational numbers, and Real numbers.

- There are some Rational numbers that are not Integers, and there are some Integers that are not Whole numbers.

- Take a moment and write down three other facts you can gather from the diagram, then have a tutor check them.

Fact 1:

Fact 2:

Fact 3:

EXERCISE 1. (ANSWER YES OR NO)

(1) Is the number 3 a rational number? **Yes, it can be written as** $\mathbf{3 = \frac{3}{1}}$

(2) Is the number 3.5 a rational number? **Yes, it can be written as** $\mathbf{3.5 = 3 + 0.5 = 3 + \frac{1}{2} = \frac{6}{2} + \frac{1}{2} = \frac{7}{2}}$**, so we have an integer 7 on top and integer 2 on bottom where the bottom integer is not 0.**

(3) Is the number -3 a whole number? **No, it is a negative integer.**

(4) Is the number 3 a natural number? **Yes**

(5) Is the number -3 an integer? **Yes**

(6) Is the number -3 a rational number? **Yes, it can be written as** $\mathbf{-3 = -\frac{3}{1}}$**.**

(7) Is the number -7 a natural number?

(8) Is the number $\frac{1}{2}$ an integer?

(9) Is the number 0 a whole number?

(10) Is the number $\frac{6}{1}$ a rational number?

(11) Is the number -7 a rational number?

(12) Is the number 1.5 rational number?

(13) Is the number 0 a natural number?

(14) Is the number 100 an integer?

(15) Is the number 101 a whole number?

(16) Is the number $-\frac{23}{3}$ a rational number?

(17) Is the number 0 a rational number?

(18) Is the number -1.5 a negative integer?

(19) Is the number $\frac{1}{2}$ a natural number?

(20) Is the number 3.7 an integer?

(21) Is the number -3 a whole number?

(22) Is the number 3.7 a rational number?

(23) Is the number -3.7 a rational number?

(24) Is the number -99 a positive integer?

(25) Is the number 1021 a natural number?

(26) Is the number $\frac{4}{5}$ an integer?

(27) Is the number 1 a whole number?

(28) Is the number -1 a rational number?

(29) Is the number 1021 a rational number?

(30) Is the number $\frac{6}{1}$ a real number?

(31) Is the number 0 a real number?

THE REAL NUMBER LINE

The real numbers can be visualized with a **real number line**. The real number line consists of a horizontal line that is cut into two parts by a point in the middle called the origin. We label the origin with the number 0. Numbers to the left of 0 are negative and numbers to the right of 0 are positive. The number 0 is neither positive nor negative, and the numbers extend leftward and rightward indefinitely, as shown in Figure.

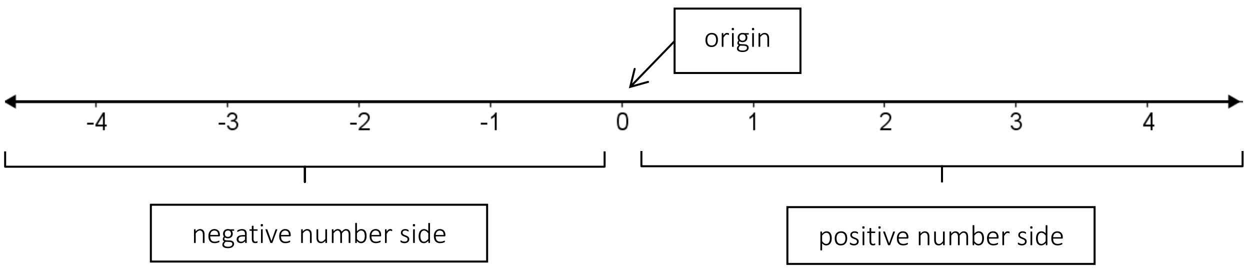
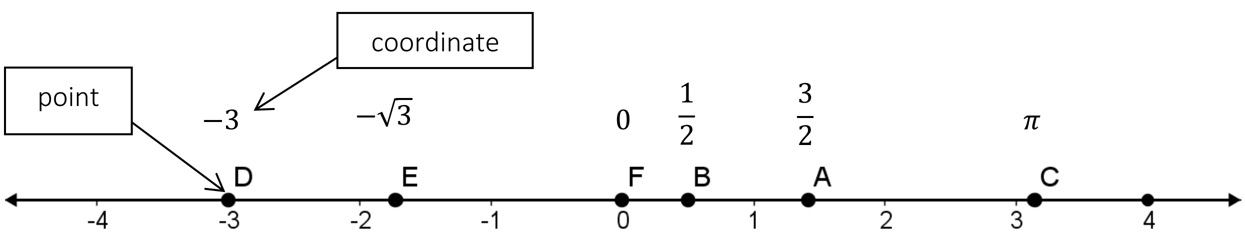


Figure. The real number line

We **plot** points on the real number line that correspond to real numbers by placing a dot approximately where the number would lie relative to a uniform scale. We say that each point on the number line corresponds to exactly one number and that each number corresponds to exactly one point. We label points with capital letters and associate a number to each point called the coordinate of the point.

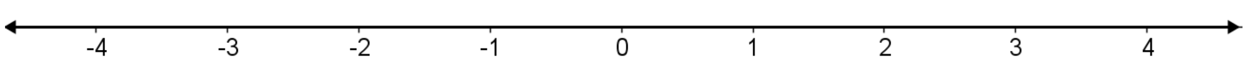


Notice that the number line includes points corresponding to all the sets of numbers. An interesting fact is that between the point corresponding to 0 and the point corresponding to 1 there lies an infinite number of points. While points corresponding to irrational numbers may seem few and far between this is not true for there are more irrational numbers than rational in the set of real numbers.

EXERCISE. Plot points on a number line that correspond to the following set of numbers.

$$\{\, 0, \frac{1}{4}, -3, 4, \pi, \sqrt{2}, -\frac{3}{2}, \frac{3}{3} \}$$

Label the point with a capital letter and place the coordinate (the number) above the point



ABSOLUTE VALUE

*absolute value = distance that a number is from 0

*Looks like: |3|
Read like: "the absolute value of 3"

*If you see |3|, the vertical bars around 3 tell you to ask the question, **"How far is 3 from 0?"**

*If you see |-3|, you should ask, **"How far is negative 3 from 0?"**

***Notice**: Absolute value is **always positive**. It is not possible to have a negative distance.

***Notice**: |3| = 3 and |-3| = 3, so if you are asked what numbers have an absolute value of 3**, you need two numbers**.

***Notice**: |3| = -3 is not possible because all absolute values are greater or equal to 0.

***Notice:** Add and subtract inside of an absolute value first, then apply absolute value operation. For example, $|4+6| = |10| = 10$ and $|10-5| = |5| = 5$.

EXERCISE 2. (DETERMINE THE ABSOLUTE VALUE)

(1) $|-\mathbf{5}| = \mathbf{5}$

(2) $|\mathbf{0}| = \mathbf{0}$

(3) $\left|-\frac{\mathbf{1}}{\mathbf{2}}\right| = \frac{\mathbf{1}}{\mathbf{2}}$

(4) $|-\boldsymbol{\pi}| = \boldsymbol{\pi}$

(5) $|\mathbf{1.5}| = \mathbf{1.5}$

(6) $|-\mathbf{5}| - |-\mathbf{5}| = \mathbf{5} - \mathbf{5} = \mathbf{0}$

(7) $|4| =$

(8) $|-4| =$

(9) $|-10.4| =$

(10) $|-5| + |-5|$

(11) $|10| =$

(12) $|0| =$

(13) $|-3.14| =$

(14) $|12{,}345| =$

(15) $\left|-\frac{9}{4}\right| =$

(16) $|-2| =$

(17) $|-6| - |-5| =$

(18) $|-8| + |47| =$

(19) $|4+6| =$

(20) $|10-5| =$

EXERCISE 3. (FIND NUMBER(S) WITH FOLLOWING THE ABSOLUTE VALUES)

(21) absolute value is $\mathbf{5}$: 5, −5

Notice: |5| = 5, |-5| = 5

(22) absolute value is $\mathbf{4.7}$: 4.7, −4.7

(23) absolute value is $-\mathbf{5}$:

$\boldsymbol{absolute\ value\ cannot\ be\ negative}$

(24) absolute value is $\frac{\mathbf{1}}{\mathbf{2}}$: $\frac{\mathbf{1}}{\mathbf{2}}, -\frac{\mathbf{1}}{\mathbf{2}}$

(25) absolute value is 4 :

(26) absolute value is $56{,}789$:

(27) absolute value is 100 :

(28) absolute value is 3.14 :

(29) absolute value is $\frac{23}{19}$:

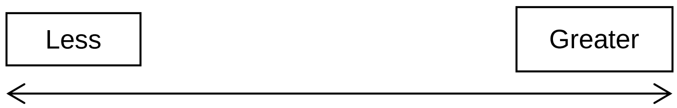
(30) absolute value is -23 :

(31) absolute value is 1 :

(32)	absolute value is 0 :

COMPARING NUMBERS

*Greater numbers are to the right and less numbers are to the left.



*Use inequality symbols $<, >, \leq, \geq$ to compare numbers.

COMPARISON SYMBOLS (WITH EXAMPLES)
$\boldsymbol{a} = \boldsymbol{b}$ **means "a is equal to b"** $3 = 3$ means "three is equal to three".
$\boldsymbol{a} \neq \boldsymbol{b}$ **means "a is not equal to b"** $-3 \neq 3$ means "negative three is not equal to three".
$\boldsymbol{a} < \boldsymbol{b}$ **means "a is less than b"** $-3 < 3$ means "negative three is less than three".
$\boldsymbol{a} > \boldsymbol{b}$ **means "a is greater than b"** $-3 > -100$ means "negative three is greater than negative one hundred".
$\boldsymbol{a} \leq \boldsymbol{b}$ **means "a is less than or equal to b"** $-3 \leq -3$ means "negative 3 is less than or equal to negative 3 ", and this is true because they are equal. <u>Only one of the two "<" or "=" has to be true</u>.
$\boldsymbol{a} \geq \boldsymbol{b}$ **means "a is greater than or equal to b"** $3 \geq 0$ means "three is greater than or equal to zero".

*__Notice:__ Be careful when comparing negative numbers. If we have -40 > -100, this is true because -100 is to the left of -40 on the number line.

*__Notice:__ Remember the number on the <u>left</u> is considered <u>lesser</u>.

*__Notice:__ You may need to change from a decimal to a fraction to make the appropriate comparison.

EXERCISE 4. (INSERT APPROPRIATE COMPARISON SYMBOL TO MAKE A TRUE STATEMENT.)

(1) $\boldsymbol{-\frac{1}{2} = -\frac{7}{14}}$

(2) $\boldsymbol{-7.23 < -6.23}$

(3) $\boldsymbol{101.01 > 101}$

(4) $\boldsymbol{67 < 167}$

(5) $\boldsymbol{4 = \frac{4}{1}}$

(6) $\boldsymbol{-\frac{3}{2} = -1.5 < -1}$

(7) -1 _____ 0

(8) -1111 _____ -1110

(9) -1.25 ____ -1.5

(10) 100 _____ 100.0001

(11) 3 _____ -3

(12) $-\frac{1}{3}$ _____ $-\frac{2}{3}$

(13) -3.56 ____ -4.56

(14) 0 _____ -0.002

(15) 234 _____ 134

(16) $-\frac{2}{3}$ _____ $-\frac{6}{9}$

(17) 1 _____ $\frac{3}{2}$

(18) 0 _____ $\frac{1}{45}$

(19) -11 _____ -12

(20) $\frac{6}{1}$ _____ 6

(21) -1.75 _____ $-\frac{3}{2}$

(22) 0 ____ k $(for$ all $integers$ $k > 1)$

WRITE T IF TRUE, BUT IF FALSE, REWRITE WITH CORRECT SYMBOL.

(23) $0 > -1$

(24) $-1212 < -121$

(25) $-0.5 > 0.25$

(26) $101 > 100.999$

(27) $-0.001 < 0$

(28) $-31 > -30$

(29) $0 \leq 34$

(30) $1.5 > \frac{7}{4} = 1.75$

(31) $4 > k \; (for\ all\ integers\ k < 2)$

INTRODUCTION TO EXPONENTS

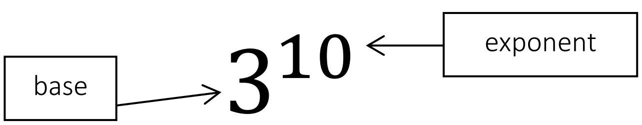
If we would like to add together the same number many times we introduce multiplication and then we can write in a more **compact manner** such as,

$$\underbrace{3+3+3+3+3+3+3+3}_{8} = 3 \times 8.$$

With this idea of "**compacting**" in mind let us look at multiplication. If we want to multiply the same number, say 3, two times, we write 3×3. Now if we want to repeat multiplication by 3 many times, we can write it in the following **compact** way (Let us say 10 times.),

$$\underbrace{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}_{10} = 3^{10}.$$

We have special names for 3 and 10.



The **base** is the number being multiplied, and the **exponent** tells you how many times to multiply the base with itself.

ORDER OF OPERATIONS FOR EXPRESSIONS

*__Order of Operations__ = steps used to combine numbers when applying more than one operation

ORDER OF OPERATIONS (ABBREVIATION AT ENDS)

Step 1: Perform operations in parentheses (par.)

Step 2: Calculate numbers with exponents (exp.)

Step 3: Multiply and divide from left to right (m.d.)

Step 4: Add and subtract from left to right (a.s.)

EXERCISE 5. (EVALUATE THE EXPRESSION. PLACE THE ABBREVIATION AT END TO EMPHASIZE YOUR PROCESS.)

(1)

$$\begin{aligned}(5-2)^2+7 &= 3^2+7 \ \ (par.)\\ &= 9+7 \ \ (exp.)\\ &= 16 \ \ (a.s)\end{aligned}$$

(2)

$$\begin{aligned}3+[3+(7-3)]^2 &= 3+[3+4]^2 \ (par.)\\ &= 3+7^2 \ (par.)\\ &= 3+49 \ (exp.)\\ &= 52 \ (a.s.)\end{aligned}$$

(3)

$$\begin{aligned}3 \times 4 \div 2 - 3 \times 4 \div 2 &= 12 \div 2 - 12 \div 2 \ (m.d.)\\ &= 6-6 \ (m.d.)\\ &= 0 \ (a.s.)\end{aligned}$$

(4)

$$\begin{aligned}4^2 \div 8 \times (2^2+4^2) &= 16 \div 8 \times (4+16) \ (exp.)\\ &= 16 \div 8 \times 20 \ (par.)\\ &= 2 \times 20 \ (m.d.)\\ &= 40 \ (a.s.)\end{aligned}$$

(5) $4+(5-3) =$

(6) $4^2 - 3 \times 2 =$

(7) $(4-1)^2 + 4 \div 2 =$

(8) $4^2 \times 2^2 - 4^2 \div 2 + 1 =$

(9) $4^2 - 3 \times 4 \div 2 =$

(10) $(5-2)^3 + 1 \times 6 =$

(11) $(5-2) \times (3^2 - 7) =$

INTEGERS AND OPPOSITES

*Integers = { ... -3, -2, -1, 0, 1, 2, 3, ... }

*Integers include:

Positive Integers = {1, 2, 3, ... }
Negative Integers = {-1, -2, -3, ... }
and the number 0

*The Integers are infinite.

*The opposite of 3 is -3, and read "negative three".

*The opposite of -3 is +3, and read "positive three" or just "three".

***Notice:** The opposite of a negative is positive and the opposite of a positive a negative.

*Integers can model different situations.

debt of 3 dollars: -3
gain of 3 dollars: 3
no loss and no gain: 0
3 feet above sea level: 3
3 feet below sea level: -3

EXERCISE 6. (WRITE POSITIVE OR NEGATIVE. THEN WRITE THE OPPOSITE INTEGER)

(1) 5 is a positive integer. The opposite is -5.

(2) -4 is a negative integer. The opposite is 4.

(3) -43

(4) 43

(5) 0

(6) -1

(7) -100

(8) 100

(9) 32

(10) -82

WRITE AN INTEGER THAT MODELS THE SITUATION

(11) profit 200 dollars from selling antique plates

(12) owe grandma 30 dollars from poker game

(13) you are on the surface of a pond in a boat (0 ft)

a. an old tire is 5ft below you

b. an air balloon is 200ft above you

(14) 4 units to the left of 0 on the number line

(15) 6 units to the right of 0 on the number line

(16) It is very cold, dang, 10 below 0

(17) You are in a bike race, and 3 miles in front of the 2nd place rider

(18) You are the 2nd place rider, and 3 miles behind

ADDING INTEGERS

*Think of the positive integers and negative integers as two types of numbers, so you are working with a negative-type and a positive-type.

*If you add $\boxed{only}$ positive-type integers you get a positive-type integer as a result.

For example, 2 and 3 are positive, and 2 + 3 = 5 where 5 is positive.

*If you **add** $\boxed{only}$ <u>negative-type integers</u> you get a <u>negative-type integer</u> as the result.

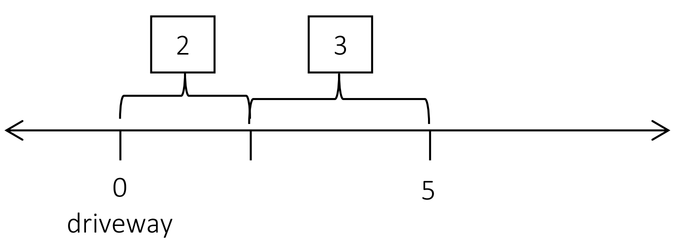
For example, -2 and -3 are negative, and -2 + (-3) = -5 where -5 is negative.

*Now if you **add** a positive-type integer with a negative-type integer, they will cancel each other (somewhat like subtraction), and the "larger number" (that is, the number with the greater absolute value) will determine if the result is negative or positive.

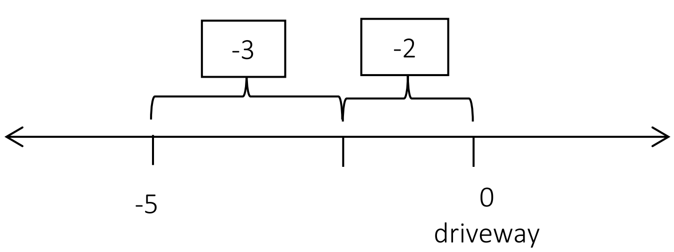
For example, **-8 is a negative-type** and **4 is a positive type**, and -8 + 4 = -4. Notice that there are more negative-types than positive-types so a negative-type results, or |-8| = 8 is greater than |4| = 4, so the absolute value of negative-type wins out, and the result it negative.

DEMONSTRATION. (IMAGINE THE FOLLOWING SITUATION.)

Imagine that you are at end of your **driveway** facing the road, at 0 steps from your driveway. Now you can go **left or right**. Say you go **right** 2 steps, which we can model with the integer 2. Now say you take 3 more steps in the same direction, so you are now 5 steps from your driveway, which we can model as 5, or 2 + 3 = 5. Here is a picture.

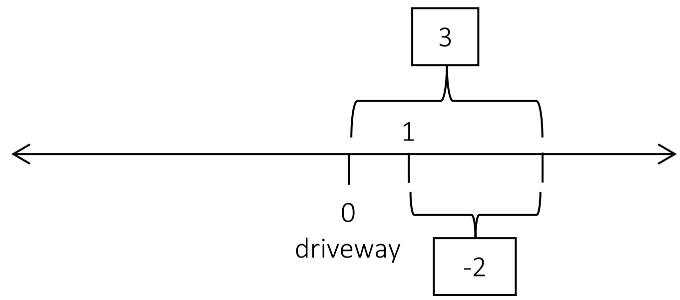


Now imagine you are at your driveway, so at 0. Now you go **left** 2 steps, which we can model as -2. Then take 3 more steps leftward, modeled as, -3. So you are now 5 steps away from your driveway but on the left side , so we model this as -5, or -2 + (-3) = -5.

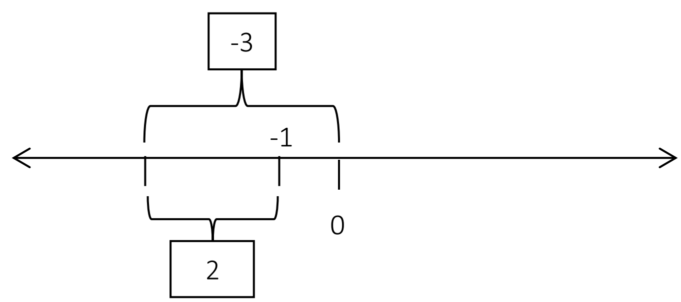


Notice: The **negative-type** corresponds to the **leftward** movement and **positive-type** corresponds to the **rightward** movement.

Ok, let us hover back to the driveway at 0. Now you take 3 steps to the right, then turn around and take two steps back toward the left. Where are you? At 1, right. We can model this situation by using **both positive-type and negative-type integers**, that is, 3 + (-2) = 1.



Hover back. Now from the driveway move 3 leftward, so you are at -3. Now move 2 rightward. Where are you? This movement can be modeled as -3 + 2 = -1, so you are -1 steps from your driveway. Here is a picture.



EXERCISE 7. (ADD THE INTEGERS)

(1) **3 + 1 = 4**

(2) **-1 + 3 = 2**

(3) **-3 + 1 = -2**

(4) **-3 + (-1) = -4**

(5) -5 + 0 =

(6) -5 + (-7) =

(7) -3 + (-55) =

(8) 23 + 53 =

(9) 23 + (-53) =

(10) -23 + (-53) =

(11) -23 + (-53) + 7 =

(12) -23 + (-53) + (-7) =

(13) 0 + (-32) =

(14) 40 + 5 + 2 =

(15) -40 + 5 + 2 =

(16) -40 + (-5) + (-2) =

(17) -30 + 10 + (-5)+ (-5) =

(18) 1 + 1 + 2 + (-1) + (-1) =

SUBTRACTING INTEGERS

*Subtracting positive integers is the same as adding a positive and negative integer.

For example, 3 - 2 = 3 + (-2) = 1.

*Subtracting negative integers is a bit different.

*Look at 3 - (-1) = 4.

***Notice:** We start with 3, subtract (-1) and get 4. This suggests that **subtracting a negative-type number is like adding a positive-type of the same absolute value.**

***Quick technique:** When you see an expression similar to 3 - (-1) do this:

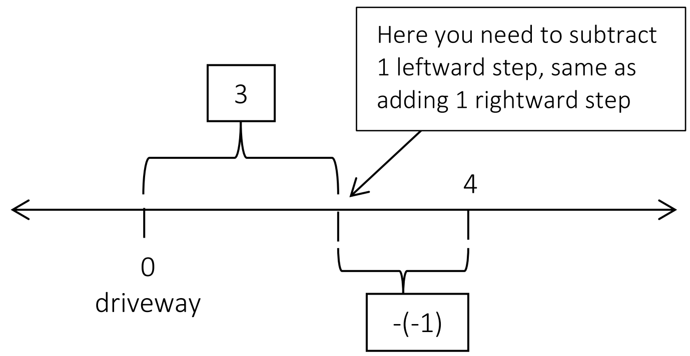
Extend the "-" across the parenthesis to create "+"

3-(-1) = 3 ~~-(-~~1)= 4.

Then add as you normally would.

*Let us use the "driveway" demonstration to explain subtracting negative integers.

Imagine you are at your driveway. You walk 3 steps right, or 3. Now you turn to walk 1 step left because the 1 is of the negative-type, that is, -1. But the -1 is subtracted, so at three steps away from your driveway **you are 1 step too far left**, so you have to step backward 1 step, which is the same as walking forward one step in the positive direction. You can think of **subtracting a negative-type integer as taking away leftward steps**, which is equivalent to adding rightward steps. Here is a picture.



EXERCISE 8. (SUBTRACT THE INTEGERS)

(1) 3 - 4 = 3 + (-4) = -1

(2) 6 - 20 = 6 + (-20) = -14

(3) -3 - 4 = -3 +(-4) = -7

(4) 3 - (-4) = 3 + 4 = 7

(5) -3 -(-4) = -3 + 4 = 1

(6) 6 -20 - (-6) = 6 + (-20) + 6 = -14 + 6 = -8

(7) 1 - 5 =

(8) 1 - (-5) =

(9) 13 - 5 =

(10) -13 - 5 =

(11) -13 - (-5) =

(12) 12 - 4 =

(13) -12 -4 =

(14) 12 - (-4) =

(15) -12 - (-4) =

(16) 15 - 10 - 5 =

(17) 15 - (-10) - 5 =

(18) 15 - (-10) - (-5) =

(19) 1 - (-2) - (-3) - (-4) =

(20) -1 - 2 - 3 - 4 =

(21) 10 - 20 - 30 - (-40) =

(22) -15 - 15 - (-10) - 21 =

(23) 0 - 5 =

(24) 0 - (-5) =

MULTIPLYING INTEGERS

*Multiplication can be expressed in the three following ways:

$$3 \times 2, \quad 3 \cdot 2, \quad or \ 3(2).$$

The x, dot, and parentheses all mean the same thing, "times".

*A **positive-type times a positive-type** is normal multiplication, so you get a positive-type integer.

For example 3(3) = 9, and 9 is positive.

*For a **positive-type integer times a negative-type**, the result is a **negative type**.

For example, 3(-2) = -6. This can be written as add -2 with itself 3 times, or (-2) + (-2) + (-2) = -6. Think of this as taking 2 left steps from your driveway 3 times, which puts us at -6 steps from the driveway.

*For a negative-type integer times a negative-type, the result, surprisingly, is a positive-type.

For example, (-3)(-2) = 6.

***Notice (Very Important):** A negative plus a negative is still negative but a negative times a negative is positive.

For example, -3 + (-4) = -7 and (-3)(-4) = 12.

*Summary:

1. (positive)times(positive) = positive
2. (negative)times(positive) = negative
3. (positive)times(negative) = negative
4. (negative)times(negative) = positive

*Let us look at multiplying more than 2 integers.

For example,

(2)(3)(4)(5) = 120	(no negatives)
(-2)(3)(4)(5) = -120	(1 negative, odd)
(-2)(-3)(4)(5) = 120,	(2 negatives, even)
(-2)(-3)(-4)(5) = -120	(3 negatives, odd)
(-2)(-3)(-4)(-5) = 120	(4 negatives, even)

***Notice:** If the number of negatives is **0 or even**, then the resulting number is **positive**. Even natural numbers are: 2, 4, 6, 8, 10, ...

***Notice:** If the number of negatives is **odd**, then the result is **negative**. Odd natural numbers are: 1, 3, 5, 7, ...

*Look at the following example and determine whether it is positive or negative,

(-1)(-1)(-1)(-1)(-1)(-1)(-1)(-1)(-1)(-5).

Any number times 1 is itself, so the answer is 5, but is it negative or positive. Let us count the number of " - " signs. There are nine in front of the 1's and one in front of the 5, so a total of 10. 10 is even, so the answer is positive 5, or just 5.

(-1)(-1)(-1)(-1)(-1)(-1)(-1)(-1)(-1)(-5) = 5.

***Quick Technique**:

Step 1: Do not pay attention to " - " signs, and **just multiply numbers**.

Step 2: **Count " - " signs**.

(even or 0) means (positive)

(odd) means (negative)

EXERCISE 9. (MULTIPLY INTEGERS)

(1) 3(4) = 12

(2) -3(4) = -12

(3) -3(-4) = 12

(4) -3(-4)(2) = 24 (even number of " - ")

(5) -3(-4)(-2) = -24 (odd number of " - ")

(6) 5(3) =

(7) 5(-3) =

(8) -5(-3) =

(9) -5(3) =

(10) -5 + 3 =

(11) 5(3)(2) =

(12) 5(3)(-2) =

(13) 5(-3)(-2) =

(14) -5(-3)(-2) =

(15) 4(-3)(2) =

(16) 0(-3) =

(17) 0 + (-3) =

(18) -1(-1)(-1)(-3)(-5) =

(19) 34(-3) =

(20) -34(-3) =

(21) -34 + (-3) =

(22) 2(-2)(2)(2)(-2) =

(23) 10(-24)(-1) =

(24) 10 + (-24) + (-1) =

DIVIDING INTEGERS

*Division can be expressed in the three following ways:

$$3 \div 2, \qquad 3/2, \qquad \frac{3}{2}.$$

*The ÷ (called an obelus), the slash, and horizontal bar with the numbers stacked, all mean the same thing, "divided by". We also read $\frac{3}{2}$ as a fraction, that is, three-halves, without applying division.

*The numbers are divided as usual, but we have to be careful with the negative sign " - ". Luckily, we treat the " - " sign exactly like we did with multiplication.

*If we count zero or an even number of negatives, the resulting number is positive, and if we count an odd number of negatives, the resulting number is negative. **The only difference is that in some of the problems there will be " - " signs on top and bottom rather than on a single line. So we count horizontally and vertically.**

*In division as well as multiplication, the negative sign " - " is allowed to wander around. To see this we can write " - " sign as (-1), so for instance -3 = (-1)3.

Let us look at examples of this "wandering nature".

(Reminder: The chain of equal signs just means that all of these expressions mean the same thing, that is, they are all equal to each other.)

Example. (-3)(2) = (-1)(3)(2) = (3)(-1)(2) = (3)(-2).

***Notice**: Even though the negative sign " - " moves around and even changes to (-1), the above expressions are all equal because there is only one negative sign.

Example.

$$(-4) \div 2 = 4 \div (-2) = -\frac{4}{2} = \frac{-4}{2} = \frac{4}{-2} = -2$$

***Notice**: The negative sign " - " can wander all around, as long as you **do not increase or decrease** the number of " - " signs. Now this only works for multiplication and division, **not** addition or subtraction.

Example. Accidently increasing by one negative sign. (≠ means "not equal")

$$-\frac{4}{2} \neq \frac{-4}{-2} \xLeftrightarrow{after\ dividing} -2 \neq 2$$

***Notice**: On the left of ≠ there is only one negative, and on the right there are two, so the two numbers are not equal. After dividing it is easy to see.

Example.

$$\frac{-4}{-2} = 2$$

***Notice:** We count the negative signs, and see that there are two, which is even, so after dividing 4/2, we know the number is positive, so we don't add a negative sign.

Example.

$$\frac{-(-(-4))}{-2} = 2.$$

***Notice:** Let us approach with two steps**. First, deal with the numbers**, so 4/2 = 2. **Second, count the negative signs**; there are 4 (even), so the result is positive 2.

EXERCISE 10. (DIVIDE THE INTEGERS)

(1) $100 \div (-25) = -4 \quad (Note: \frac{100}{25} = 4\ and\ one\ -)$

(2) $-55 \div (-11) = 5 \quad (Note: \frac{55}{11} = 5\ and\ two\ -)$

(3) $8 \div (-4) =$

(4) $(-8) \div (-4) =$

(5) $(-16) \div 4 =$

(6) $54 \div (-9)$

(7) Are all of these equal (y or n)? $\frac{-24}{2} = -\frac{24}{2} = \frac{24}{-2}$

(8) Are all of these equal (y or n)? $-\frac{24}{-2} = \frac{24}{2} = -12$

(9) $-\frac{-12}{-2} =$

(10) $\frac{81}{-9} =$

(11) $-\frac{56}{7} =$

(12) $\frac{-32}{-4} =$

Hint for 13: Divide numbers first, then count negative signs to determine whether positive or negative.

(13) $\frac{-(-21)}{-(-(-3))} =$

PUTTING IT ALL TOGETHER

*Let us practice with

(1) sets
(2) absolute value
(3) comparing numbers
(4) exponents
(5) order of operations
(6) adding, subtracting, multiplying, and dividing integers

EXERCISE 11. (ANSWER THE FOLLOWING PROBLEMS)

(1) List the sets that the number -3 does not belong within?

(2) Write -5 as a rational number.

(3) Find a number that is in the integers, but not in the negative integers, and not in the natural numbers.

(4) Is that number greater than -3?

(5) Suppose today you write the number **0.3**, then every hour for the rest of your life you add another 3 to the end of the number, like 0.333... . Then after you pass, your child does the same, then their child, and so on. Your number is now 0.3333...333...and on!

a. Will the number ever become bigger than the number 1? Explain.

b. After each 3 is added will that number still be a real number? A rational numbers? An integer? Explain.

c. Do you know what number your generations of family members are working towards (Hint: It is a fraction with 1 on top and an odd number less than 10 on the bottom, try a few quotients on a calculator)?

COMPARE OR CALCULATE.

(1) |23| =

(2) |-12,345| =

(3) $\frac{1}{2}$ ____ $\frac{1}{3}$

(4) |4| ____ |-4|

(5) |0.12|____|-0.13|

(6) $(6 \div 2)^3 - 3 \times 2 + 7 =$

(7) $24 \div 2 \div 3 \div 2 \times 2 - 4 =$

(8) 23 + (-5) =

(9) 23 - (-5) =

(10) 23 + (-5) - (-7) =

(11) 23 + (-5) - (-7) - 4 + (-2) =

(12) 23(-5) =

(13) (-23)(-5) =

(14) (-3)(-6)(2)(-4) =

(15) $\frac{-2(-8)}{-4} =$

(16) $\frac{2(9)}{3(-2)} =$

(17) $|2^2 - 12|$ ____ $|(3-1)^2 - 2(6)|$

(18) |-32 - 4| ____ |50 + (-13)|

(19) Is the following true? $\left|-\frac{56}{7} - (-9)\right| < |-2|$

WHAT ARE ALGEBRAIC EXPRESSIONS?

*You have already worked with normal **expressions**. All of the following are expressions:

23 + (-5) $\qquad (6 \div 2)^3 - 3 \times 2 + 7 \qquad \frac{81}{-9}$

*algebraic expression = an expression with unknown numbers called variables.

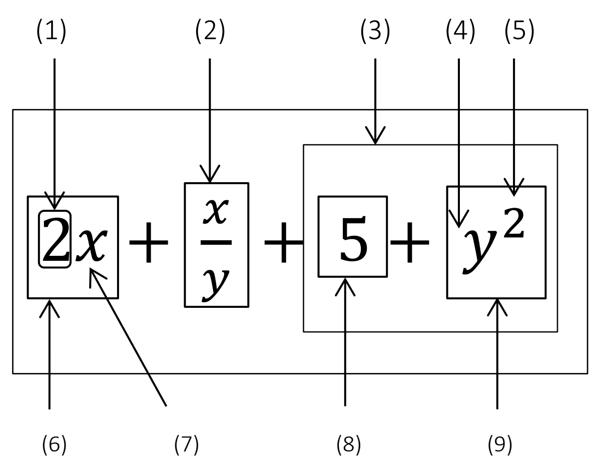
- Use letters like **x, y, z, m, n, a, b** as variables.

*We can make our above expressions algebraic by taking out numbers and replacing them with variables x, y, z.

x + (-5) $\qquad (y \div 2)^3 - x \times 2 + z \qquad \frac{y}{-9}$

*Parts of an algebraic expression have special names. The following example introduces these names.

$2x + \frac{x}{y} + 5 - y^2$ is an algebraic expression.



(1) **Coefficient**. Number in front of the variable.

(2) **Term**. Compare with (6). Not a monomial because quotient of variables.

(3) **Sub-expression**.

(4) Imaginary 1: If no coefficient, imagine there is a 1 in front of the variable, so $(1)y^2$ in this case.

(5) Variable **exponent**. Remember y is just a number.

(6) **Term**. Also called **monomial** because product of variables. <u>All constants are considered monomials</u>.

(7) **Variable**. Stands for a number. Also called **unknown**.

(8) **Constant** Term. Only a number. <u>Also monomial</u>.

(9) Term with invisible coefficient 1. Also monomial.

<u>More Examples</u>.

-Constant Terms (no variables): $4,\ 0.5,\ \frac{17}{2},\ -3.3,\ \ 1$

-Terms: $2x,\ -4x^2,\ \frac{1}{2}y,\ 3.14xy,\ \ y$

. . . . .

-Coefficients (from Terms): $2,\ -4,\ \frac{1}{2},\ 3.14,\ \ 1$

*The term, 3.14xy, is a monomial because it is a **product of variables.**

*The term, $\frac{3.14x}{y}$, is <u>not</u> a monomial because it has a **quotient of variables**.

EXERCISE 12.(STRIP APART THE FOLLOWING ALGEBRAIC EXPRESSIONS AND PLACE PARTS IN APPROPRIATE ROW. MAY HAVE REPEATS)

(1) $3x - 5 - xy - 7 = \mathbf{3x + (-5) + (-1)xy + (-7)}$
(Hint: Write subtraction as adding negative integers to make it easier to pick out coefficients and constant terms)

Term: **3x, -5, -xy, -7**	Monomial: **3x, -5, -xy, -7**
Coefficient: **3, -1**	Constant: **-7, -5**

(2) $4.5x - 7$

Term:	Monomial:
Coefficient:	Constant:

(3) $\frac{2}{3}x - 7y + \frac{x}{y}$
(Hint: Watch for quotients of variables. Not a monomial.)

Term:	Monomial:
Coefficient:	Constant:

(4) $-4x^2 + 2ab + 6.22$

Term:	Monomial:
Coefficient:	Constant:

(5) $4x + x^2 + \frac{1}{2} - x + 3$

Term:	Monomial:
Coefficient:	Constant:

<u>WRITING ALGEBRAIC EXPRESSIONS</u>

*When solving real-life or story problems you will have to take verbal expressions and write them as algebraic expressions to make the problem easier to solve.

Words and phrases associated with operations		
Operation	**Verbal expression**	
<u>Addition</u> $x + y$	sum of x and y total of x and y x plus y x increased by y	gain raise x more than y increase of
<u>Subtraction</u> $x - y$	<u>Difference of x and y</u> x minus y x decreased by y x subtracted by y	loss fewer take away x less y
<u>Multiplication</u> xy	Multiply x by y Product of x and y x times y	double x twice x triple x
<u>Division</u> $\frac{x}{y}$	quotient of x and y divide x by y x divided by y divided equally	per <u>x separated into y equal parts</u>

***Notice:** If you are not given variables then introduce x, y, z, m, n, a, b, c, or any other symbol you would like, to stand for the number(s).

For example, say you were given the verbal expression "**the sum of two numbers**". To start, introduce two symbols to stand as those numbers. Write,

Let a and b be two numbers.

Then you can write the verbal expression as,

a + b

Now suppose you heard the verbal phrase "a number increased by 5". Here you only have to introduce one symbol because the other number, namely 5, was given. So you can write,

Let x be a number

Then the verbal expression written as an algebraic expression is

x + 5

If you would prefer, you could first write,

number + 5

then **x + 5**

***Notice:** The symbol is only a placeholder for a number, so you could even use ☺ to stand for a number. You could write x + 5, as

☺ + 5

EXERCISE 13.(INTRODUCE VARIABLES (SYMBOLS) WHERE NECESSARY AND WRITE VERBAL EXPRESSIONS AS ALGEBRAIC EXPRESSIONS)

(1) the sum of two numbers
let m, n be numbers. m + n
(2) the sum of two number multiplied by 4
let a,b be numbers. (a + b)(4)
(3) the difference between two numbers

(4) the product of three numbers

(5) the quotient of two numbers

(6) 3 more than k

(7) b decreased by a number

(8) divide a number by another number

(9) a number increased by 121

(10) 4 less than a number

(11) 5 times a number

(12) r plus a number

(13) multiply a number by 5 then divide by 6

(14) m separated into 4 equal parts

(15) raise a number by a product of two distinct numbers

(16) a number subtracted by the quotient of 2 and y

(17) a number decreased by a product of two distinct numbers

(18) the sum of double a number and triple a different number

(19) a number decreased by m plus n

EVALUATING EXPRESSIONS USING SUBSTITUTION

*substitution = the act of plugging a known number in for a variable

*Algebraic expressions contain an **unknown number** which we call a **variable**. The **variable** acts as a placeholder and allows us to substitute into the expression almost any number, and then do a calculation.

*evaluate = the act of substituting variables with given numbers and calculating the expression

Example. Suppose we have the following algebraic expressions.

(1) $x - y$

(2) $4x + 5y$

(3) $\frac{x^2}{3} + \frac{y^2}{2}$

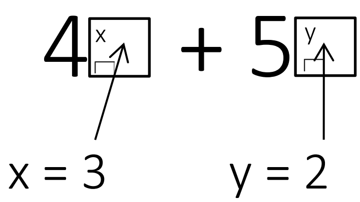
Now let x = 3 and y = 2. Then by substituting (plugging) in 3 for all the x's and 2 for all the y's we can evaluate the expressions.

(4) $x - y = \boxed{3 - 2} = 1$

(5) $4x + 5y = \boxed{4(3) + 5(2)} = 12 + 10 = 22$

(6) $\frac{x^2}{3} + \frac{y^2}{2} = \boxed{\frac{3^2}{3} + \frac{2^2}{2}} = \frac{9}{3} + \frac{4}{2} = 3 + 2 = 5$

More clearly, think of it like this.



Notice you have an x-box and a y-box. Put the 3 into the x-box because x = 3, then put the 2 into the y-box because y = 2.

$$4\boxed{3} + 5\boxed{2}$$

Then multiply and add to get 22.

EXERCISE 14. (EVALUATE THE EXPRESSIONS WITH THE GIVEN NUMBERS)

FOR (1) - (9), Let x = 2, y = 3, a = -2,

(1) x + y + a **= 2 + 3 + (-2) = 5 + (-2) = 3**

(2) 2x + 3y - a **= 2(2) + 3(3) - (-2) = 4 + 9 + 2 = 15**
(Hint: Be sure to include the negative sign when substituting.)

(3) x + 3 =

(4) x + y + 3 =

(5) $x^2 + y + 3a =$

(6) $\frac{x+y}{5} + a =$

(7) $\frac{2x+2y}{5} =$

(8) $\frac{2x+x}{a} =$

(9) $\left(\frac{xya}{2}\right)(y - x) =$

FOR (10) - (15), Let x = 4, y = -1, a = 2

(10) x + y + 3 =

(11) $x^2 + y + 3a =$

(12) $\frac{x+y}{5} + a =$

(13) $\frac{2x+2y}{3} =$

(14) $\frac{2x+x}{a} =$

(15) $\left(\frac{xya}{2}\right)(y - x) =$

ADDING AND SUBTRACTING LIKE TERMS

*Like Terms = **terms** in an expression that have the exact same variables with the exact same exponents

*Combine Like Terms = you can add and subtract **like terms**

*Do you remember what a term is?

*In 3x + 4y, **3x and 4y are terms**, but they are not "like terms".

*The following table shows examples of "like terms". **The entries across a row are considered "like".** Look for a pattern.

$2x \longrightarrow$	$-4x \longrightarrow$	$100x$	$0.5x$	$\frac{1}{2}x$
$2x^2 \longrightarrow$	$-4x^2$	$100x^2$	$0.5x^2$	$\frac{1}{2}x^2$
$2xy$	$-4xy$	$100xy$	$0.5xy$	$\frac{1}{2}xy$
$2x^2y$	$-4x^2y$	$100x^2y$	$0.5x^2y$	$\frac{1}{2}x^2y$
$2x^2y^5$	$-4x^2y^5$	$100x^2y^5$	$0.5x^2y^5$	$\frac{1}{2}x^2y^5$

***Notice:** Across each row, the variables and their exponents are exactly alike.

*Lets construct an expression from some terms in the table.

$$\boxed{100x + (-4x)} + \boxed{2x^2 + 0.5x^2} + 100x^2y$$

*We can **combine like terms** once we determine which terms are alike. Let us do that.

100x and -4x have x in common so they are **like terms**.

$$100x + (-4)x = 96x$$

$2x^2$ and $0.5x^2$ have x^2 in common so they are **like terms**.

$$2x^2 + 0.5x^2 = 2.5x^2$$

Then putting it together we get,

$$100x + (-4x) + 2x^2 + 0.5x^2 + 100x^2y = \boxed{96x + 2.5x^2 + 100x^2y}$$

The terms in the box are not alike, so we are done.

*<u>simplified</u> = the expression is simplified when you have combined all the like terms.

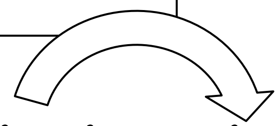
***Notice:** Be sure you recognize that x and x^2 are <u>not</u> alike. <u>Remember the exponent has to be the same</u>.

*<u>**The Formal Rule and the Cartoon Way**</u>

It is important to understand the **formal** rule that allows us to combine like terms, but sometimes a **cartoon** analogy can make things easier.

<u>**Formal rule**</u>: Use the **distributive property** to factor out common terms then simplify.

The distributive property says that we can factor out a common term x^2



$$2x^2 + 0.5x^2 = (2 + 0.5)x^2$$
$$= 2.5x^2$$

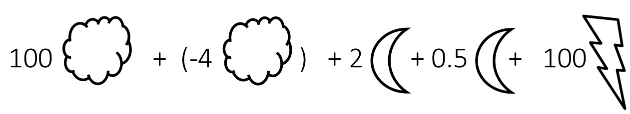
We will cover the distributive property more in depth when we are multiplying algebraic expressions.

<u>Cartoon Way</u>: Let us turn it into a cartoon.

Suppose we have

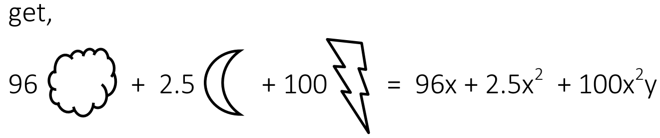
$$100x + (-4x) + 2x^2 + 0.5x^2 + 100x^2y$$

So we have **x, x^2, and x^2y as variables of the terms**. Let us replace them with cartoons.



We can combine clouds with clouds, moons with moons, and bolts with bolts but we can't combine moons with clouds or any differing cartoons. They have to be alike.

We have 100 clouds but we need to take away 4, so we get 96 clouds. We have 2 moons and 0.5 moons, so we get 2.5 moons, and we have 100 bolts, and that is it. We get,



The key idea here is to treat the variables as little pictures, go through the expression find similar pictures, and add up or subtract the numbers in front. Remember that the numbers in front are called coefficients.

Example.

Formal way:

$$\begin{aligned} 3x + (-3)ab + 2x + 5ab &= 3x + 2x + (-3)ab + 5ab \\ &= (3+2)x + (-3+5)ab \\ &= 5x + 2ab \end{aligned}$$

After moving terms around we apply the distributive property twice then combine coefficients.

Cartoon way:
Think of the variables as little drawings not letters.

$$3x + (-3)ab + 2x + 5ab$$

We have two different drawings: **x** and **ab**.
There are 3 x's, then 2 more, so 5 total x's.
There are -3 ab's, then 5 more, so 5 take away 3, that is 2 ab's. Then together

~~3x~~ + ~~(-3)ab~~ + ~~2x~~ + ~~5ab~~ = 5x + 2ab.

As you move through longer expressions strike out or underline terms that you have already combined so you can keep track of where you are at.

***Remember:** 3 - (-2) means 3 + 2, that is, a minus-minus becomes a plus.

EXERCISE 15. (COMBINE LIKE TERMS TO SIMPLIFY THE EXPRESSION. SOME EXPRESSIONS MAY ALREADY BE SIMPLIFIED.)

(1) $2x + 3x =$

(2) $2x - x =$

(3) $2x - (-x) =$

(4) $3c + 2c =$

(5) 3 ☺ - 2 ☺ =

(6) $3abc - 2abc =$

(7) $-3abc + 2abc - (-3)ac =$

(8) $2x + 3x + 2x =$

(9) $2x + 3x + 2y =$

(10) $2x - 3y + 2y =$

(11) $2x - 3y -2y =$

(12) $2x + (-3)y + (-2)y =$

(13) $2x^2 + 2xy =$

(14) $2x^2 + 2y^2 + 1x^2 =$

(15) $3a - (-4)a =$

(16) $3a - 4a =$

(17) $-3ab - 2ab + a =$

(18) $2x^2y + 21x^2y - 4x^2y =$

(19) $\frac{1}{2}x + \frac{1}{2}x =$

(20) $0.3y + 0.7y + 0.5xy =$

(21) $-3fz - 2fz + z + (-3)z =$

(22) $\frac{1}{2}x + \frac{1}{2}x + \frac{1}{2}x + \frac{1}{2}x =$

(23) $\frac{1}{2}x - \frac{1}{2}x + \frac{1}{2}x - \frac{1}{2}x =$

(24) $\frac{1}{2}x - \frac{2}{3}y + \frac{1}{2}x - \frac{2}{3}y =$

(25) $-k^2 + 2k^2 + 3k^3 + 4k^2 =$

(26) $0.5omg + 0.25omg + 0.1om =$

(27) $0.5om^2g - 0.25omg - 0.1om =$

(28) $4k + (-4)kt + 5k + 5kt =$

(29) $5r - 4r - r - (-5)r =$

(30) $-21c + 3c^3 - 4c^3 =$

(31) $1a + 2ab + 3abc + 4abcd + 5abcde =$

(32) 2abcde + 3abcde - (-3)abcde + 10abde =

MULTIPLYING MONOMIALS

*Remember monomials:

$2x \quad 3xy \quad 5 \quad x^2 \quad 7a \quad \frac{1}{2}bc$

Recall they are a product (multiplication) of numbers and variables.

*A quotient of variables is **not** a monomial. So,

$$\frac{x}{y} \qquad \frac{ab}{c} \qquad \frac{1}{x}$$

are not monomials.

*We can combine monomials by **multiplication.** Study the table and try to see the pattern.

Monomials		Multiplication
6x	3y	(6x)(3y) = 6(3)xy = 18xy
3y	-3	(3y)(-3) = 3(-3)y = -9y
8ab	-2c	(8ab)(-2c) = 8(-2)abc =-16ab
6ab	2a	(6ab)(2a) = 6(2)aab = $12a^2b$

***Notice:**
1. You are allowed to move factors around.(commute and associate).

2. You need to get the numbers and variables together.

3. Multiply the numbers and put the variables (letters) in alphabetical order.

4. If there are copies like **a(a)** you can write with an exponent, so

$$a(a) = a^2$$

***Notice**: We are associating numbers and commuting numbers. Since variables are really numbers we can also associate and commute variables.

To associate: Study the example.

$$(2 \cdot 3) \cdot 4 = 2 \cdot (3 \cdot 4)$$
$$6 \cdot 4 = 2 \cdot 12$$
$$24 = 24$$

To associate means that we can multiply the 2 and 3, then multiply by 4, or can multiply 3 and 4 first, then multiply by 2. The result should be the same.

To commute: Study the example.

$$3 \cdot 4 = 4 \cdot 3$$
$$12 = 12$$

To commute means that we can flip or change the order of the numbers and we will get the same result.

***Commuting and associating** allow us to take numbers and variables in a larger product, move them around, and simplify the expression. For example,

$$e \cdot 3 \cdot c \cdot 5 \cdot a \cdot 4 \cdot b \cdot (-2) \cdot d$$
$$= (3 \cdot 5 \cdot 6 \cdot (-2)) \cdot a \cdot b \cdot c \cdot d \cdot e$$
$$= -180 \cdot a \cdot b \cdot c \cdot d \cdot e$$

EXERCISE 16. (MULTIPLY THE FOLLOWING MONOMIALS)

(1) (18)a = **18a**

(2) a(a) = $\mathbf{a^2}$

(3) a(a)(a) = $\mathbf{a^3}$

(4) -4(3) =

(5) -4(3x) =

(6) -4(-3x) =

(7) -4a(3x) =

(8) 4a(-3y) =

(9) 5xyz(-2abc)

(10) 0.5(-4x) =

(11) -0.3(-3w)

(12) $-x(-4y) =$

(13) 4a(2b)(3c) =

(14) 4a(-2b)(-3c) =

(15) -3a(4) =

(16) (-3rs)(-2t)(5ab) =

(17) x(x) =

(18) x(x)(x) =

(19) 2x(x)(x) =

(20) 2x(-3x)(y) =

(21) 0(2x) =

(22) $33(\frac{1}{7}y)$ =

(23) $\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)x$ =

(24) 0.2a(0.3s)(0.4d) =

(25) (4 · 5) · bnm =

(26) -3j(4jk)2k =

(27) a(-5b)(2d)(3c)(2ae) =

DIVIDING MONOMIALS

*We can combine monomials <u>by division</u>. Study the table of examples to introduce yourself to the process.

Monomials		Division
6x	3y	$\frac{6x}{3y} = \left(\frac{6}{3}\right)\left(\frac{x}{y}\right) = 2\left(\frac{x}{y}\right) = \frac{2x}{y}$
3y	-3	$\frac{3y}{-3} = \left(\frac{3}{-3}\right)y = \left(\frac{3}{-3}\right)\left(\frac{y}{1}\right) = (-1)y = -y$
8ab	-2c	$\frac{8ab}{-2c} = \left(\frac{8}{-2}\right)\left(\frac{ab}{c}\right) = (-4)\left(\frac{ab}{c}\right) = -\frac{4ab}{c}$
6ab	2a a≠ 0	$\frac{6ab}{2a} = \left(\frac{6}{2}\right)\left(\frac{a}{a}\right)\left(\frac{b}{1}\right) = 3 \cdot 1 \cdot b = 3b$

The process of <u>division</u> is very similar to multiplication.

1. You are allowed to **<u>move factors around</u>** as long as you keep them on the same level, that is, no top factors can go to the bottom, and no bottom factors to the top.

***Remember:** $-\frac{1}{1} = \frac{-1}{1} = \frac{1}{-1} = -1$ are all different ways to write the same value.

***Tip:** If you have $3\left(\frac{4x}{3}\right)$, put a 1 under the 3, so you know that it is on top (in the numerator).

$$3\left(\frac{4x}{3}\right) = \left(\frac{3}{1}\right)\left(\frac{4x}{3}\right) = \frac{3 \cdot 4x}{1 \cdot 3} = \frac{12x}{3}$$

***Remember**: $3 = \frac{3}{1}$ $\quad and \quad$ $-3 = -\frac{3}{1}$.

2. **Get numbers and variables together.**

3. **Divide numbers and put variables in alphabetical order on each level.**

4. **Write copies as exponents that are on same level, otherwise if on opposite levels, cancel (by dividing).**

$$\frac{a \cdot b \cdot b}{a} = \frac{a \cdot b^2}{a} = \left(\frac{a}{a}\right)b^2 = 1 \cdot b^2 = b^2$$

***Notice:** a is just a number, and if you divide a number by itself you get 1. For example, $\frac{3}{3} = 1$, so $\frac{a}{a} = 1$.

<u>We will not worry about this right now</u>, but mention that when a = 0, we cannot divide a by itself because $\frac{0}{0}$ is undefined. <u>Any number divided by zero is undefined</u>.

MAJOR RULE: You cannot divide by 0.

You could try to divide by zero, but it just does not work in the real numbers, sort of like a human being trying to fly with only their arms.

EXERCISE 17. (DIVIDE THE MONOMIALS. ASSUME NO VARIABLES EQUAL 0)

(1) $\frac{6x}{2} =$

(2) $\frac{-6a}{3} =$

(3) $\frac{6x}{-2} =$

(4) $-\frac{18kj}{2} =$

(5) 2x + (-4x) =

(6) $(4)\left(\frac{6xyz}{2}\right) =$

(7) $\frac{8}{4}ab =$

(8) $\frac{8}{4}a(-b) =$

(9) -3ab - (-y) + 5ab + y =

(10) $\frac{6a}{2b} =$

(11) $\frac{-a}{5}(5c) =$

(12) $\frac{(-x)(10gt)}{2} =$

(13) $\frac{x}{x} =$

(14) $\frac{64x}{2x} =$

(15) $\frac{56gh}{-7} =$

(16) $\left(\frac{6x}{2}\right)\left(\frac{3y}{3}\right) =$

(17) $\frac{100 \cdot a \cdot b \cdot b}{-2a}$

THE DISTRIBUTIVE PROPERTY

*The distributive property allowed us to combine like terms. For example,

$$3x + 2x = (3 + 2)x = 5x.$$

You can see we pulled out the common factor, namely x, then added the numbers.

*The distributive property allows us to do two actions:

Action 1: Factor

a. First look at each term and see if there is a common factor that appears in each term. Here it is an x.

b. Pull the x from the terms, called factoring, then put it out in front of sum (y + 3).

$$\boxed{x} \cdot y + \boxed{x} \cdot 3 = \boxed{x} \cdot (y + 3)$$

Action 2: Distribute

a. Notice below you have a number x times a sum (y + 3).

b. Apply the x to each of the terms in sum (in the parentheses), called distributing. You are "distributing the x".

$$x \cdot (y + 3) = x \cdot y + x \cdot 3$$

The distributive property is very important because it will allow you to solve equations. .

EXERCISE 18. (USE THE DISTRIBUTIVE PROPERTY AS DIRECTED)

Factor and combine if possible.

(1) 3x + 4x = (3 + 4)x =7x (x is common)

(2) $3xy + 4xy = (3+4)xy = 7xy$ (xy is common)

(3) $3x + yx = (3+y)x$ (x is common)

(4) $4x + 4y = 4(x+y)$ (4 is common)

(5) $4x + 4y + 4z = 4(x + y + z)$ (4 is common)

(6) $6a + 3a =$

(7) $6ab + 3ab =$

(8) $6a + ba =$

(9) $7t - rt =$

(10) $5a + 5b =$

(11) $5a + 5b + 5c =$

(12) $-14a - 14b =$

(13) $4a + 3a =$

(14) $\frac{1}{2}a + \frac{1}{2}b =$

(15) $0.5j + 0.5k + 0.5a =$

(16) $ab + bc =$

(17) $-ab - bc =$

(18) $3ab - 4bc =$

(19) $3ab - 3bc =$

Distribute and combine if possible.

(1) $2(x + y) = 2x + 2y$ (distribute 2 to x and y)

(2) $-2(x + y) = -2x + (-2y) = -2x - 2y$ (distribute -2)

(3) $x(y + z) = xy + xz$ (distribute x to y and z)

(4) $3(a + b) =$

(5) $-3(a+b) =$

(6) $3(2a + 3b + c)=$

(7) $-3(2a + 3b) =$

(8) $-3(3a) =$

(9) $a(4 + 6) =$

(10) $a(b + c) =$

(11) $a(2b + 4c) =$

(12) $2a(2b - 4c) =$

(13) $2a(2b + (-4c)) =$

(14) $3jkb(a + d) =$

(15) $4g(a + b + c + 2) =$

(16) $-3(-a - b - c) =$

(17) $-4bc(b + c) =$

(18) $3(k + -3n + \frac{1}{3}j + t^2) =$

(19) $3p(k + -3n + \frac{1}{3}j + t^2) =$

ADDING AND SUBTRACTING FRACTIONAL EXPRESSIONS

*fractional expression = expression with numbers and variables in fractional form.

*Suppose you have the fraction $\frac{1}{3}$. Now replace 1 with the variable a, and you have $\frac{a}{3}$. This is a **fractional algebraic expression**.

*A quotient of monomials, previously studied, is a fractional expression.

*We can add and subtract fractional expressions.

Study the table of examples.

$\boldsymbol{x+\frac{1}{2}x} = x\cdot 1+\frac{1}{2}x = x\left(\frac{2}{2}\right)+\frac{1}{2}x = x\left(\frac{2}{2}+\frac{1}{2}\right) = \frac{3}{2}x$
$\boldsymbol{\frac{x}{2}-\frac{x}{3}} = \frac{x}{2}\cdot\frac{3}{3}-\frac{x}{3}\cdot\frac{2}{2} = \frac{3x}{6}-\frac{2x}{6} = \frac{x}{1}\left(\frac{3}{6}-\frac{2}{6}\right) = \frac{x}{6}$
$\boldsymbol{a-b+\frac{c}{2}} = \frac{a}{1}\left(\frac{2}{2}\right)-\frac{b}{1}\left(\frac{2}{2}\right)+\frac{c}{2} = \frac{2a-2b+c}{2}$
$\boldsymbol{\frac{2a}{3}+\frac{4a}{7}} = \left(\frac{7}{7}\right)\left(\frac{2a}{3}\right)+\left(\frac{3}{3}\right)\left(\frac{4a}{7}\right) = \frac{14a}{21}+\frac{12a}{21} = \frac{26a}{21}$ find common denominator   combine top terms

***Notice**: When adding fractional expressions, we must get a common denominator, exactly the same process as when combining fractions. Here is an example. Remember?

$$\frac{1}{4}+\frac{1}{3} = \left(\frac{3}{3}\right)\left(\frac{1}{4}\right)+\left(\frac{4}{4}\right)\left(\frac{1}{3}\right) = \frac{3}{12}+\frac{4}{12} = \frac{7}{12}$$

***Notice**: After getting a common denominator in both terms, we combine the top terms (terms in the numerators).

EXERCISE 19. (ADD OR SUBTRACT)

(1) $p+\frac{p}{2}=$

(2) $\frac{p}{2}+\frac{p}{3}=$

(3) $\frac{4a}{3}+\frac{a}{3}=$

(4) $\frac{t}{7}-\frac{k}{7}=$

(5) $\frac{5a}{2}+\frac{4a}{3}-\frac{2}{6}=$

(6) $3-\frac{b}{2}=$

(7) $\frac{2e}{2}+e-3e=$

(8) $\frac{p}{2}-\frac{p}{3}=$

(9) $\frac{pj}{2}-\frac{5p}{3}=$

(10) $\frac{x}{4}+2x-\frac{x}{5}=$

(11) $\frac{x}{4}+2x-\frac{x}{5}+y=$

(12) $r+s+\frac{t}{3}=$

(13) $\frac{7t}{7}+\frac{6t}{6}-\frac{5t}{5}=$

(14) $-4g+\frac{7g}{1}=$

(15) $\frac{0}{5}-\frac{w}{5}+\frac{4w}{5}=$

(16) $\frac{sw}{3}+\frac{5sw}{4}=$

(17) $\frac{p}{2}+\frac{p}{3}+\frac{p}{4}-\frac{13p}{12}=$

MULTIPLYING FRACTIONAL EXPRESSIONS

*We can also multiply fractional expressions.

- As you look through the table of examples notice these three things:

1. We can unify the top factors into a single product as well as the bottom.
2. We can divide by numbers directly below or at angles. You can rearrange (associate and commute) factors to make the division and finding fractional equivalents (reduction of fractions) more clear.
3. After dividing and reducing we multiply across just as we would with a regular fraction.

TABLE OF EXAMPLES

$\left(\frac{a}{2}\right)\left(\frac{b}{3}\right) = \frac{ab}{2 \cdot 3} = \frac{ab}{6}$
$\left(\frac{6a}{1}\right)\left(\frac{b}{3}\right)\left(\frac{c}{6}\right) = \frac{\cancel{6}abc}{1 \cdot 3 \cdot \cancel{6}} = \frac{abc}{3}$
$\frac{-3xy}{10} \cdot \frac{10b}{1} \cdot \frac{c}{-3} = \frac{\cancel{-3} \cdot xy \cdot \cancel{10} \cdot bc}{\cancel{10} \cdot 1 \cdot \cancel{-3}} = xybc$

multiply

1   -5

$$\frac{3t}{4} \times \frac{-20}{9s} = \frac{\cancel{3} \times t \times (-\cancel{20})}{\cancel{4} \times \cancel{9} \times s} = \frac{-5t}{3s}$$

1   3

multiply

Above we created a unified product on top and bottom, reduced the fraction 3/9 to 1/3, and divided -20 by 4, then multiplied across.

$$\frac{3t}{4} \times \frac{-20}{9s} = \left(\frac{3}{4}\right)\left(-\frac{20}{9}\right)\left(\frac{t}{1}\right)\left(\frac{1}{s}\right) = \left(-\frac{20}{4}\right)\left(\frac{3}{9}\right)\left(\frac{t}{s}\right)$$

$$= \left(-\frac{5}{1}\right)\left(\frac{1}{3}\right)\left(\frac{t}{s}\right) = -\frac{5t}{3s}$$

Here we approached the same problem but differently. We pulled the coefficients to the front. Rearranged top and bottom, reduced and divided, then multiplied across.

Study both and decide which way you find easier. Also, decide whether you prefer writing multiplication with a dot, an x, or parentheses. Be familiar with each but pick one to use in your work, so you don't mix things up.

EXERCISE 20. (MULTIPLY FRACTIONAL EXPRESSIONS. HINT: REDUCE AND DIVIDE NUMBERS BEFORE MULTIPLYING ACROSS)

(1) $\left(\frac{a}{1}\right)\left(\frac{b}{2}\right) =$

(2) $\frac{2a}{1} \cdot \frac{b}{2} =$

(3) $\frac{2a}{3} \times \frac{3b}{2} =$

(4) $\left(\frac{2}{a}\right)\left(\frac{3}{b}\right) =$

(5) $\frac{2a}{1} \cdot \frac{b}{26} \cdot \frac{13}{c} =$

(6) $\frac{2a}{3} \times \frac{-9k}{4} =$

(7) $\left(\frac{2}{3a}\right)\left(\frac{3}{2b}\right)\left(-\frac{c}{5}\right) =$

(8) $\frac{-9a}{8} \cdot \frac{-72v}{-81} =$

(9) $\frac{2a}{3} \times \frac{3b}{2} \times \frac{7c}{14} =$

(10) $\left(\frac{2a}{-3}\right)\left(\frac{27}{bk}\right)\left(\frac{c}{18}\right) =$

(11) $\frac{2a}{1} \cdot \frac{b}{2} \cdot \frac{3}{2} \cdot \frac{2}{3e} =$

(12) $-\frac{121a}{11} \times \frac{33b}{11} \times \frac{-c}{-33} =$

(13) $-3a(4bc)(-2)\left(\frac{x}{24}\right) =$

(14) $\frac{2a}{4} + \frac{5a}{2} - \frac{4a}{8} =$

(15) $\frac{2a}{3} \times \frac{3b}{2} \times b \times \frac{a}{2} =$

PUTTING IT ALL TOGETHER

We have covered:

Sets of Numbers	Absolute Value	Comparing Numbers
Exponents	Order of Operations	Add, Subtract, Multiply, Divide Integers
Components of Algebraic expressions	Evaluating Expressions	Add, Subtract, Multiply, Divide (Monomial)Terms
Distributive Property	Adding and Subtracting Fractional Expressions	Multiplying Fractional Expressions

EXERCISE 21. (ANSWER THE FOLLOWING. LOOK BACK AT PREVIOUS SECTIONS IF YOU GET STUCK.)

(1) Take numbers from the set and list them next to the set in which they belong. You will repeat numbers.

$$\left\{0, -3, \sqrt{2}, \pi, -\frac{1}{4}, \frac{34}{3}, 34, 10, 202, -0.01, 1\right\}$$

natural numbers:

whole numbers:

negative integers:

integers:

rational numbers:

irrational numbers:

(2) Order the set of numbers **from greatest to least** using the inequality symbol >. For example, similar to

5 > 4 > 3 > 2 > 1.

$$\left\{0, 1, -3, -0.001, 0.5, -\frac{1}{2}, 10, 9.99, -3.01, -\frac{1}{4}\right\}$$

(3) $|[3 - (21 - 2 \cdot 7)^2] \div 2| =$

(4) $|100 \div 2 \div 25 \div 2 \times 3 - 3| =$

(5) $1 - \big(3 - (-5)(2)\big) =$

(6) $\frac{4}{-2} + \frac{6}{-2} + \frac{8}{-2} + \frac{10}{-2} - (-14) =$

(7) $\frac{3\left(4-(-(-2))\right)}{-6} =$

(8) Is $\frac{x}{yz}$ a monomial?

(9) What are the (a) coefficients of the terms and (b) how many terms are there?

$$\frac{1}{2}x^3 + 0.5x^2 - 2xy + 3x$$

(a)

(b)

EVALUATE.

(10) Let $a = 5,\ b = -3,\ c = -\frac{1}{2}$.

$$\big(a - (-b)\big) \cdot c =$$

COMBINE.

(11) $(3a - 2b) + (5a - 4b) =$

(12) $3x + (-3x + 2y) + (-2y + k) =$

(13) $2(a + b) + 3(a + b)$

(14) $3(x + 2) + (-3x) =$

(15) $4x + \frac{1}{2}x - 0.5y - (-2y) =$

(16) $\frac{2x}{4} + \frac{3y}{2} - \frac{3a}{6} =$

(17) $2x - \left(-\left(-(-x)\right)\right) =$

(18) $\frac{1}{2}(2x - 8y) =$

(19) $\frac{1}{3}(-9a - 18b) + \frac{1}{5}(5a + 25b) =$

(20) $\frac{a}{1} \cdot \frac{b}{3} \cdot \frac{3c}{4} \cdot \frac{4d}{5} \cdot \frac{5e}{6} \cdot \frac{12}{2} =$

WHAT ARE EQUATIONS?

*An equation is a mathematical statement that says two expressions have the same value.

For example, say we have the following two simple expressions:

3x and 9

Even though we do not know what x is we can assume that whatever it is, 3x has the same value as 9. We write,

3x = 9.

We can take the two expressions, 3x and 9, and construct other equations, for example,

3x + 3x + 3x = 9
Or
9 + 3x = 3x + 9(3x)

*In the box above, we are only playing with symbols (making things up as we go) to explore the concept of an equation, but later you will actually take a real-world problem and model it with an equation to derive a real-world solution.

***Notice**: We simply have a left-hand-side and a right-hand-side,

LHS = RHS

***What do we do with an equation?**

We solve equations.

Look at 3x = 9. This equation begs the question, "3 times what number equals 9?" That is not too bad; we find x should be 3 because 3(3) = 9.

Look at 3x + 3x + 3x = 9. This equation begs the question, "Three times what number plus 3 times that same number again plus three times that same number again equals 9?" You can probably figure it out, but you can see that equations become complex very easily, so it is best that we find a sure way to answer the question, "What is x?" without melting our minds.

***How do we solve harder equations?**

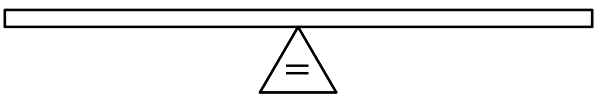
The following sections of this book will instruct methods to help you with solving equations.

Let us look quickly at a very important but simple tool that you will use over-and-over when solving equations. For our purposes, we will call it the Balance Rule.

Balance Rule: What you do to one side of the equation, you must do to the other side of the equation to keep balance (also called equality).

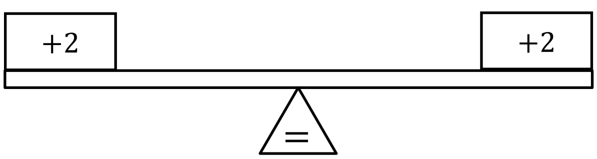
DEMONSTRACTION. BALANCE RULE

(1)



Suppose picture (1) represents the equation **x - 2 = 9**. We assume that it is balanced when it is first given as a problem.

(2)



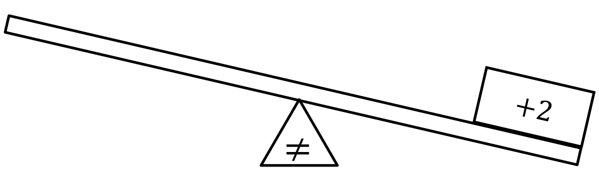
In picture (2) we add +2 to both sides of the equation, so we have

$$x - 2 \underline{+2} = 9 \underline{+2}$$

By the Balance Rule our equation is still balanced because we added 2 to both sides, that is, what we did to one side, we did to the other. If we combine numbers we get,

$$x = 11$$

(3)



In picture (3) we added +2 to the RHS but not the LHS, so our equation is

$$x - 2 = 9 + 2.$$

But our equation is no longer balanced because we did not add +2 to the LHS.

If we solve for x now we will find **x = 13.**

Our original equation was **x - 2 = 9.**

When **x = 11**, we get

$$x - 2 = 11 - 2 = 9 = 9$$

which is what we want.

When **x = 13**, we get

$$x - 2 = 13 - 2 = 11 \neq 9.$$

But this not good because 11 is not 9.

It is very important to use the **Balance Rule** or else your solution may not satisfy the equation.

When we say "**satisfy the equation**" we mean that when we take our solution, such as x = 11, and substitute (plug) it into the original equation, the LHS and the RHS should be equal (the same).

You should always plug your solution back into the original equation to make sure your solution procedure is correct, that is, **always make sure your solution satisfies the equation.**

***Conclusion:** If you add a number or variable to one side, do the same to the other side to get the correct solution.

You can do the same for subtraction, multiplication, and division. Here are some examples where the **Balance Rule** has been applied correctly.

$$x + 2 = 3 \quad \xLeftrightarrow[+4]{\text{do same to both sides}} \quad x + 2 + 4 = 3 + 4$$

$$x + 2 = 3 \quad \xLeftrightarrow[-4]{\text{do same to both sides}} \quad x + 2 - 4 = 3 - 4$$

$$x + 2 = 3 \quad \xLeftrightarrow[\times 4]{\text{do same to both sides}} \quad 4(x + 2) = 4 \cdot 3$$

$$\boldsymbol{x + 2 = 3} \quad \xLeftrightarrow[\div 4]{\textbf{do same to both sides}} \quad \frac{\boldsymbol{x+2}}{4} = \frac{\boldsymbol{3}}{4}$$

EXERCISE. (USE THE ABOVE EXAMPLE TO GUIDE YOUR WORK. DERIVE AN EQUATION THAT IS BALANCED WHERE YOU START WITH $x+1=5$.)
$x+1=5 \quad \overset{\text{do same to both sides} \; +7}{\Longleftrightarrow}$
$x+1=5 \quad \overset{\text{do same to both sides} \; -7}{\Longleftrightarrow}$
$x+1=5 \quad \overset{\text{do same to both sides} \; \times 7}{\Longleftrightarrow}$
$\boldsymbol{x}+1=5 \quad \overset{\textbf{do same to both sides} \; \div 7}{\Longleftrightarrow}$

Before we start to solve equations, there is one more general rule that you can follow to guide your work. For our purposes we will call it the **Isolation Rule**.

***Isolation Rule**: Use the Balance Rule and what you know from combing expressions **to isolate** the variable on one side of the equation, leaving the numbers on the other side.

Here is an example.

$$\begin{aligned} x-2&=9 \\ x-2+2&=9+2 && (Balance\text{: }add\ 2\ on\ \ sides) \\ \boldsymbol{x+(-2+2)}&\boldsymbol{=11} && (Combine\ integers) \\ x+0&=11 && (Combine\ integers) \\ x&=11 && (x\ is\ isolated\ on\ one\ side) \end{aligned}$$

SOLVING ADDITION AND SUBTRACTION EQUATIONS

*A simple **addition equation** looks like: x + 3 = 9

*A simple **subtraction equation** looks like: x - 3 = 9

*These two equations are easy to solve. What plus 3 equals 9? We know, 6. For the other equation, What minus 3 equals 9? Not too bad; the answer is 12.

*The equations become less direct when we include negative integers and negative variables. For example, try to solve x - (-3) = 9 on the fly.

VERY IMPORTANT: YOUR GOAL IS TO ISOLATE THE VARIABLE ON ONE SIDE OF THE EQUAL SIGN LEAVING ALL THE NUMBERS ON THE OTHER SIDE.

*Study these examples.

$\boxed{r+5=61}$ $r+5-5=61-5$ $r+(5-5)=56$ $r+0=56$ $r=56$ 56 is the solution	$\boxed{a+(-5)=-73}$ $a+(-5)+5=-73+5$ $a+0=-68$ $a=-68$ -68 is the solution
$\boxed{g-4=-9}$ $g-4+4=-9+4$ $g+0=-5$ $g=-5$ -5 is the solution	$\boxed{x-(-3)=9}$ $x+3=9$ $x+3-3=12-3$ $x+0=9$ $x=9$ 9 is the solution

***Notice:** On the LHS of each equation there is a variable and a number. Observe we **took the opposite** of that number and applied it to both sides to isolate the variable.

***Notice:** When we had x - (-3) we rewrite the LHS as x+3 before solving.

***Notice:** We can start with 61 = r + 5 instead of r + 5 = 61. Same equation.

***Notice:** It is good practice to rewrite the equations in a vertical stream, aligning the equal sign as you solve, but sometime it is easier to cancel with a vertical method. For example, we can solve the equation in the following way,

$$\begin{array}{r} r+5=61 \\ -5 \quad -5 \\ \hline r+0=56 \\ r=56 \end{array}$$

This method will be practical when solving multistep equations.

EXERCISE 22. (SOLVE THE EQUATIONS)	
(1) a + 3 = 6	(2) s + 23 = 23 - (-20)

(3) b - (-3) = 6	(4) 6 = c - 3
(5) c - 3 = 6	(6) 6 = b - (-3)
(7) x + (-4) = 18	(8) 6 - (-5) = 11 + x
(9) y + (-4) = -18	(10) k = 23 - (-34) - 3
(11) z - 4 = -18	(12) 2x - x =
(13) -17 + m = -44	(14) 2x - x = 5
(15) 17 - (-n) = 22	(16) t + 21 + (-3) = 4
(17) -45 + r = -22	(18) 56 - (-34) + k = 21 - (-21)
(19) s - (-34) = -34	(20) 4x - 2x - x + (-4) = -4

SOLVING MULTIPLICATION AND DIVISION EQUATIONS

*A **multiplication equation** looks like: $3x = 9$

*A **division equation** looks like: $\frac{x}{3} = 9$

*Remember what we are doing **in general**.

(1) The equation has a LHS and a RHS.

(2) We want to isolate the variable on one side.

(3) We can isolate the variable by manipulating the LHS or the RHS, but we have to be sure to use the **Balance Rule** so we don't change the equation, that is, if we multiply on one side, we have to multiply on the other.

*Before we start, let us review an old idea. Do you remember the **reciprocal**? For example,

The **reciprocal of 4** is $\frac{1}{4}$, that is, $4 \cdot \frac{1}{4} = 1.$

The **reciprocal of $\frac{1}{4}$** is 4, that is, $\frac{1}{4} \cdot 4 = 1.$

<u>The key idea</u> **is that if you have a number and multiply that number by it's reciprocal you get 1.**
Here is a table of numbers and their reciprocals. Do you see the pattern? Just flip it over to get the reciprocal.

number	reciprocal	number	reciprocal
$4 = \frac{4}{1}$	$\frac{1}{4}$	$-\frac{4}{7}$	$-\frac{7}{4}$
$\frac{1}{4}$	$\frac{4}{1} = 4$	$-\frac{100}{3}$	$-\frac{3}{100}$
$-4 = -\frac{4}{1}$	$-\frac{1}{4}$	$\frac{2}{3}$	$\frac{3}{2}$
$-\frac{1}{3}$	$-\frac{3}{1}$	$3 = \frac{3}{1}$	$\frac{1}{3}$

***Notice:** If a number is negative, then it's reciprocal is negative. Also, observe that if you multiply the number by it's reciprocal you get 1.

***Tip:** Rewrite a number as a rational number if you can't see the reciprocal. For example, 4 is the same as $\frac{4}{1}$, so we can find the reciprocal by flipping it over, getting $\frac{1}{4}$.

*Study the following examples to see how we use the reciprocal to solve multiplication and division problems. **Do you see the step where we multiply both sides of the equation by the reciprocal?**

$\boxed{5r = 30}$	$\boxed{-4a = -64}$
$\frac{5}{1}r = 30$	$-\frac{4}{1}a = -64$
multiply by reciprocal: $\left(\frac{1}{5}\right)\left(\frac{5}{1}\right)r = 30\left(\frac{1}{5}\right)$	$\left(-\frac{1}{4}\right)\left(-\frac{4}{1}\right)a = -64\left(-\frac{1}{4}\right)$
get 1: $1 \cdot r = \frac{30}{5}$	$1 \cdot a = \frac{64}{4}$
$r = 6$	$a = 16$
6 is the solution	16 is the solution

$\boxed{\frac{g}{4} = 5}$	$\boxed{\frac{x}{-3} = 9}$
$\left(\frac{1}{4}\right)g = 5$	$\frac{1}{-3}x = 9$
$\left(\frac{4}{1}\right)\left(\frac{1}{4}\right)g = 5\left(\frac{4}{1}\right)$	$\left(-\frac{3}{1}\right)\left(\frac{1}{-3}\right)x = 9\left(-\frac{3}{1}\right)$
$1 \cdot g = 5 \cdot 4$	$1 \cdot x = 9 \cdot (-3)$
$g = 20$	$x = -27$
20 is the solution	-27 is the solution

$\boxed{\frac{5g}{4} = 5}$	$\boxed{\frac{7x}{-3} = 14}$
$\left(\frac{5}{4}\right)g = 5$	$\frac{7}{-3}x = 14$
$\left(\frac{4}{5}\right)\left(\frac{5}{4}\right)g = 5\left(\frac{4}{5}\right)$	$\left(\frac{-3}{7}\right)\left(\frac{7}{-3}\right)x = 14\left(\frac{-3}{7}\right)$
$1 \cdot g = 5 \cdot \frac{4}{5}$	$1 \cdot x = 14 \cdot \left(\frac{-3}{7}\right)$
$g = 4$	$x = \left(\frac{14}{7}\right)(-3)$
4 is the solution	$x = 2 \cdot (-3)$
	$x = -6$
	-6 is the solution

***Notice:** To solve both multiplication and division problems we **multiply both sides by the appropriate reciprocal**.

While multiplying both sides by the reciprocal is the general method, we can restate the method another way.

To solve a multiplication equation **divide both sides** by the coefficient of the variable.

$$5r = 30$$

$$\frac{\cancel{5}r}{\cancel{5}} = \frac{30}{5}$$

$$r = 6$$

***Notice:** Dividing by 5 is the same as multiplying by $\frac{1}{5}$.

To solve a division equation, multiply both sides by the number beneath the variable (in the denominator).

$$\frac{g}{4} = 5$$

$$\cancel{4} \cdot \frac{g}{\cancel{4}} = 5 \cdot 4$$

$$g = 20$$

***Notice:** $4 = \frac{4}{1}$ so here we are really just multiplying by the reciprocal of $\frac{1}{4}$.

*The **benefit of multiplying by the reciprocal** is seen in the following problem.

$$\frac{7x}{-3} = 14$$

Here we have a **multiplication and division problem**. By simply multiplying by the reciprocal we can isolate x quickly

$$\downarrow \qquad \qquad \downarrow$$

$$\left(\frac{-3}{7}\right)\left(\frac{7}{-3}\right)x = 14\left(\frac{-3}{7}\right) \quad (multiply\ by\ reciprocal)$$

$$x = 14\left(-\frac{3}{7}\right)$$

We can also go about solving this problem in more steps.

$$\frac{7x}{-3} = 14$$

$$-3\ \frac{7x}{-3} = 14 \cdot -3 \qquad (multiply\ by -3)$$

$$7x = -42$$

$$\frac{7x}{7} = -\frac{42}{7} \qquad (divide\ by\ 7)$$

$$x = -6$$

EXERCISE 23. (SOLVE THE FOLLOWING EQUATIONS)	
(1) $4x = 16$	(2) $-x = 1$
(3) $\frac{a}{5} = 20$	(4) $20 = \frac{s}{5}$
(5) $2y = -8$	(6) $-8 = 2t$
(7) $\frac{b}{3} = -9$	(8) $-z = 12 + (-4)$
(9) $-5x = 45$	(10) $-9 = \frac{u}{3}$
(11) $\frac{a}{-4} = 20$	(12) $-w = 16$
(13) $-12y = -144$	(14) $\frac{2a}{5} = 20$
(15) $\frac{b}{-7} = -21$	(16) $\frac{6y}{5} = -12$
(17) $u - (-5) = 9$	(18) $-\frac{3z}{8} = 9$
(19) $-54 = -9r$	(20) $-\frac{7z}{6} = -14$

(21) $-23x = 0$	(22) $2x + 3x = 5$

SOLVING MULTISTEP EQUATIONS

-We have now solved
addition equations,
subtraction equations,
multiplication equations,
division equation.

Now we will look at more interesting equations that require more steps in the solution procedure.

Study the following **equation types** and **solution procedures.**

The work is in the left column and the explanation is in the right column

Equations with like terms	
$\boxed{3x + 4x = 14}$ $7x = 14$ $\frac{7x}{7} = \frac{14}{7}$ $x = 2$	*3x and 4x are "like" *add like terms *divide both sides by 7 *calculate
Equations with variables on both sides	
$\boxed{3x + 8 = 5x}$ $3x - 3x + 8 = 5x - 3x$ $8 = 2x$ $\frac{8}{2} = \frac{2x}{2}$ $4 = x$	*3x and 5x are on opposite sides *subtract 3x from both sides to get x's together *divide both sides by 2 *calculate

Equations involving addition and multiplication	
$\boxed{4x + 21 = 29}$ $4x + 21 - 21 = 29 - 21$ $4x = 8$ $\frac{4x}{4} = \frac{8}{4}$ $x = 2$	*need to isolate x *subtract 21 from both sides *divide both sides by 4 *calculate
Equations where a fraction equals a fraction	
$\boxed{\frac{x}{3} = \frac{1}{5}}$ $\frac{3}{1} \cdot \frac{x}{3} = \frac{1}{5} \cdot \frac{3}{1}$ $x = \frac{3}{5}$	*need to isolate x *multiply both sides by reciprocal * calculate
Equations that use the distributive property	
$\boxed{3(x + 2) = 12}$ $3 \cdot x + 3 \cdot 2 = 12$ $3 \cdot x + 6 = 12$ $3 \cdot x + 6 - 6 = 12 - 6$ $3 \cdot x = 6$ $\frac{3x}{3} = \frac{6}{3}$ $x = 2$	*need to isolate x *distribute 3 on LHS *multiply numbers *subtract 6 from both sides *divide by 3 *calculate
Equations involving fractional coefficients	
$\boxed{\frac{2}{3}x + \frac{1}{4}x = \frac{1}{6}}$ $12\left(\frac{2}{3}x + \frac{1}{4}x\right) = \frac{1}{6} \cdot 12$ $12 \cdot \frac{2}{3}x + 12 \cdot \frac{1}{4}x = \frac{1}{6} \cdot 12$ $\left(\frac{12}{3}\right) \cdot 2x + \left(\frac{12}{4}\right) \cdot 1x = \frac{12}{6}$	*need to isolate x *notice 12 is a common multiple *multiply both sides by 12 *distribute the 12 on LHS *Knock out numbers on bottom by dividing them into 12

$4 \cdot 2x + 3 \cdot 1x = 2$ $8x + 3x = 2$ $11x = 2$ $x = \frac{2}{11}$	*multiply remaining coefficients *combine like terms *divide by 11 * x is isolated

***Notice:** The above equations are solved with techniques that you have already covered. They appear more challenging because we are applying more than one of the techniques.

There is usually more than one way to solve an equation, so if you have idea, try it out, then take your solution and plug it back in to the original equation and see if it satisfies the equation.

Notice we could have solved $3(x+2) = 12$ in the following way,

$$3(x+2) = 12$$
$$\frac{3(x+2)}{3} = \frac{12}{3}$$
$$x + 2 = 4$$
$$x + 2 - 2 = 4 - 2$$
$$x = 2$$

EXERCISE 24. (SOLVE THE EQUATIONS)	
(1) $3x + 7x = 100$	(2) $-11x - 4x = 60$
(3) $3x = 100 - 7x$	(4) $2y - (-4y) = 72 - 12y$
(5) $3x + 10 = 100$	(6) $34 = 5b + (-21)$

(7) $\frac{x}{9} = \frac{14}{3}$	Hint: flip both sides (8) $\frac{1}{-27} = \frac{3}{n}$
(9) $2(x+4) = 8$	(10) $5(k+4) = 7(k+2)$
(11) $\frac{1}{2}x + \frac{1}{4}x = \frac{1}{8}$	(12) $\frac{3}{8}p - \frac{1}{4}p = \frac{2}{16}$
(13) $-3a + 7a = -100$	(14) $-200 = -3a + 7a - 14a$
(15) $-3b + 4 = 100 - 7b$	(16) $-8b + 55 = 100 - 17b$
(17) $3c + 10 = -26$	(18) $\frac{3}{2}r + 10 = -26$
(19) $\frac{24}{9} = \frac{r}{9}$	(20) $\frac{24}{9} + 1 = \frac{g}{9}$

(21) $-3(s+4)=90$	(22) $-3(t+4)+8=90$
(23) $\frac{1}{2}t+\frac{1}{4}t=\frac{2}{3}$	(24) $\frac{14}{2}w+\frac{12}{4}w=\frac{2}{3}$
(25) $3x+7x=100$	(26) $-11x-4x\ =60$

FINAL PROBLEMS

ANSWER WITH TRUE OR FALSE.

(1) All natural numbers are whole numbers.

(2) All whole numbers are natural numbers. Hint: 0.

(3) All natural numbers are rational numbers. Hint 4 = $\frac{4}{1}$

(4) At least one rational number is a natural numbers.

(5) Put in your own words the Balance Rule.

(6) Put in your own words the Isolation Rule.

MODEL WITH AN INTEGER.

(7) 20 below zero degrees

(8) 20 miles above sea level

WRITE AS AN ALGEBRAIC EXPRESSION.

(9) the sum of a and t

(10) the difference of a and t

(11) the product of a and t

(12) the quotient of a and t

(13) a number increased by the product of a and t

(14) a number decreased by the quotient of a and t

(15) the sum of the product and quotient of a and t

UNDERLINE EACH TERM AND CIRCLE THE COEFFICIENTS.

(16) $3x^2+0.5y+\left(-\frac{1}{2}\right)a$

COMPARE.

(17) -100 _______ -101

(18) |20 - 30 ÷ 10| _______ 16

EVALUATE BY SUBSTITUTION.

(19) Let a = 4

$14+a-(3a)\cdot\frac{a}{4}=$

(20) Let b = 3

$b+\frac{4b}{2}-|b|=$

CALCULATE THE EXPRESSIONS.

(21) 3^2 + 8 ÷ 4 - 16 ÷ 2 ÷ 2 =

(22) $[(-3 + -4(-4))^2 - 69] \div 2 \div 50 =$

(23) $|-3| =$

(24) $|-3(-4) + (-13)| =$

(25) 4 - (-3) + (-3) - 4 =

(26) 3(2 - 3) + (4 - 7) =

(27) $-\frac{4(3)}{2} =$

(28) $-\frac{(-3)(-21)}{-7} =$

(29) 3q + 4q - 22q =

(30) 2z + (-2z + 9a) + (-9a + 1) - 1 =

(31) -5a(-4b)(-3c)(-2d)(-1e) =

(32) $\frac{-20a(-10c)}{200} =$

(33) $\frac{-20a(-10c)}{200} + \frac{b}{100} - \frac{d}{50} =$

(34) -3(5 + a + b) =

(35) -15 - 3a - 3b =

(36) $\frac{a}{3} + \frac{a}{6} + \frac{a}{12} =$

(37) $\frac{ab}{-3} \cdot \frac{cd}{-6} \cdot \frac{216e}{12} =$

SOLVE THE EQUATIONS.	
(1) 3 + t = -21	(2) 3 - r = -21
(3) 5 - r = -21	(4) 3p = -21
(5) $\frac{x}{3} = -21$	(6) 3x + 4x = -21
(7) 3a + 4a = -21 + 4a	(8) 3x - 9 = -21
(9) $\frac{w}{3} = \frac{-42}{2}$	(10) -3(-x - 3) = -3(-3)

(11) $2\left(\frac{3}{2} + \frac{1}{2}x\right) = -21$

(12) $\frac{1}{2}x - \frac{1}{4}x = \frac{1}{8}$

(13) $2(x+3) = 3(x-3)$

(14) $\frac{11x}{5} + 2x = 25$

(15) $4(x+3) - 3(x+3) = 2x$

(16) $2\left(\frac{2x}{5} + 3x\right) + x = 2x + 4$

(17) $\frac{4x}{5} - \frac{21}{13} = 0$

SOLUTIONS TO EXERCISES

Exercise 0.

1.a. N, b. Y, c. Y
2. a. Y, b. Y, c. N, d. N
3. a. Y, b. N, c. N, d. Y

Possible examples.
Fact 1: No negative integers are rational.
Fact 2: There are real numbers are not natural numbers.
Fact 3: Real numbers contain the irrational numbers.

Exercise 1.
1. yes
2. yes
3. no
4. yes
5. yes
6. yes
7. no
8. no
9. yes
10. yes
11. yes
12. yes
13. no
14. yes
15. yes
16. yes
17. yes
18. no
19. no
20. no
21. no
22. yes
23. yes
24. no
25. yes
26. no
27. yes
28. yes
29. yes
30. yes
31. yes

Exercise 2.
1. 5
2. 0
3. 1/2
4. π
5. 1.5
6. 0
7. 4
8. 4
9. 10.4
10. 10
11. 10
12. 0
13. 3.14
14. 12,345
15. 9/4
16. 2
17. 1
18. 55
19. 10
20. 5

Exercise 3.
21. 5,-5
22. 4.7, -4.7
23. absolute value cannot be negative
24. 1/2, -1/2
25. 4, -4
26. 56,789, -56,789
27. 100, -100

28. 3.14, -3.14
29. 23/19, -23/19
30. absolute value cannot be negative
31. 1, -1
32. 0

Exercise 4.
1. =
2. <
3. >
4. <
5. =
6. <
7. <
8. <
9. >
10. <
11. >
12. >
13. >
14. >
15. >
16. =
17. <
18. <
19. >
20. =
21. <
22. <
23. T
24. T
25. F, <
26. T
27. T
28. F, <
29. T
30. F, <
31. T

Exercise 5.
1. 16
2. 52
3. 0
4. 40
5. 6
6. 10
7. 11
8. 57
9. 10
10. 33
11. 6

Exercise 6.
1. positive, -5
2. negative, 4
3. negative, 43
4. positive, -43
5. neither, no opposite
6. negative, 1
7. negative, 100
8. positive, -100
9. positive, -32
10. negative, 82
11. 200
12. -30
13. a. -5, b. 200
14. -4
15. 6
16. -10
17. 3
18. -3

Exercise 7.
1. 4
2. 2
3. -2
4. -4
5. -5
6. -12
7. -58
8. 76
9. -30
10. -76
11. -69
12. 83
13. -32
14. 47
15. -33
16. -47
17. -30
18. 2

Exercise 8.
1. -1
2. -14
3. -7
4. 7
5. 1
6. -8
7. -4
8. 6
9. 8

10. -18
11. -8
12. 8
13. -16
14. 16
15. -8
16. 0
17. 20
18. 30
19. 10
20. -10
21. 0
22. -41
23. -5
24. 5

Exercise 9.
1. 12
2. -12
3. 12
4. 24
5. -24
6. 15
7. -15
8. 15
9. -15
10. -2
11. 30
12. -30
13. 30
14. -30
15. -24
16. 0
17. -3
18. -15
19. -102
20. 102
21. -37
22. 32
23. 240
24. -15

Exercise 10.
1. -4
2. 5
3. -2
4. 2
5. -4
6. -6
7. y
8. n
9. -6
10. -9
11. -8
12. 8
13. -7

Exercise 11.
1. natural numbers, whole numbers, irrational numbers
2. -5/1
3. 0
4. yes
5.
    a. no, each three only adds a small fraction to the number
    b. yes a real number, yes a rational number, not an integer because it is a decimal fraction
    c. 1/3

1. 23
2. 12,345
3. >
4. =
5. <
6. 28
7. 0
8. 18
9. 28
10. 25
11. 19
12. -115
13. 115
14. -144
15. -4
16. -3
17. =
18. <
19. yes

Exercise 12.
1. term: 3x, -5, -xy, -7; coefficient: 3 , -1; monomial: 3x, -5, -xy, -7; constant:-7, -5

2. term: 4.5x, -7; coefficient: 4.5; monomial: 4.5x, -7; constant: -7

3. term: (2/3)x, -7y, x/y; coefficient: 2/3, -7, 1; monomial: (2/3)x, -7y; constant:

4. term: $-4x^2$, 2ab, 6.22; coefficient: -4, 2; monomial: $-4x^2$, 2ab, 6.22; constant:6.22

5. term: 4x, x^2, 1/2, -x, 3; coefficient: 4, 1, -1; monomial: 4x, x^2, 1/2, -x, 3; constant: 3, 1/2

Exercise 13.
NOTICE: Symbols will vary
1. m, n numbers, m+n
2. a, b numbers, (a+b)(4)
3. x-y
4. xyz
5. x/y
6. k+3
7. b-x
8. x/y
9. x+121
10. x-4
11. 5x
12. r+x
13. (5x)/6
14. m/4
15. a +bc
16. x - 2/y
17. x - yz
18. 2x + 3y
19. x-(m+n)

Exercise 14.
1. 3
2. 15
3. 5
4. 8
5. 1
6. -1
7. 2
8. -3
9. -6
10. 6
11. 21
12. 13/5
13. 2
14. 6
15. 20

Exercise 15
1. 5x
2. x
3. 3x
4. 5c
5. ☺
6. abc
7. -abc + 3ac
8. 7x
9. 5x + 2y
10. 2x - y
11. 2x - 5y
12. 2x - 5y
13. $2x^2 + 2xy$
14. $3x^2 + 2y^2$
15. 7a
16. -a
17. -5ab + a
18. $19x^2y$
19. x
20. y + 0.5xy
21. -5fz -2z
22. 2x
23. 0
24. x - (4/3)y
25. $3k^3 + 5k^2$
26. 0.75omg + 0.1om
27. $0.5om^2g - 0.25omg - 0.1om$
28. 9k + kt
29. 5r
30. $-21c - c^3$
31. 1a + 2ab + 3abc + 4abcd + 5abcde (no like terms)
32. 8abcde + 10abde

Exercise 16
1. 18a
2. a^2
3. a^3
4. -12
5. -12x
6. 12x
7. -12ax
8. -12ay
9. -10abcxyz
10. -2x
11. 0.9w
12. 4xy
13. 24abc
14. 24abc
15. -12a
16. 30abrst
17. x^2
18. x^3
19. $2x^3$
20. $-6x^2y$
21. 0

22. $\frac{33}{7}y$
23. $\frac{1}{16}x$
24. 0.024asd
25. 20bnm
26. $-24j^2k^2$
27. $-60a^2bcde$

Exercise 17

1. 3x
2. -2a
3. -3x
4. -9kj
5. -2x
6. 12xyz
7. 2ab
8. -2ab (it is preferred but not required to write negative sign in front)
9. 2ab + 2y
10. $\frac{3a}{b}$
11. -ac
12. -5gtx
13. 1
14. 32
15. -8gh
16. 3xy
17. $-50b^2$

Exercise 18

Note: There may be alternative factorizations.

6. 6a + 3a = (6 + 3)a = 9a
7. 6ab + 3ab = (6 + 3)ab = 9ab
8. 6a + ba = (6 + b)a
9. 7t - rt = (7 - r)t
10. 5a + 5b = 5(a + b)
11. 5a + 5b + 5c = 5(a + b + c)
12. -14a - 14b = -14(a + b)
13. 4a + 3a = (4 + 3)a = 7a
14. $\frac{1}{2}a + \frac{1}{2}b = \frac{1}{2}(a + b)$
15. 0.5j + 0.5k + 0.5a = 0.5(j + k + a)
16. ab + bc = ab + cb = (a + c)b
17. -ab - bc = -ab - cb = (-a - c)b
18. 3ab - 4bc = 3ab - 4cb = (3a - 4c)b
19. 3ab - 3bc = 3b(a - c)

1. 2x + 2y
2. -2x - 2y
3. xy + xz
4. 3a + 3b
5. -3(a + b) = -3a -3b
6. 3(2a + 3b +c) = 6a + 9b + 3c
7. -3(2a + 3b) = -6a - 9b
8. -9a
9. a(4 + 6) = a10 = 10a
10. a(b + c) = ab + ac
11. a(2b + 4c) = 2ab + 4ac
12. 2a(2b - 4c) = 4ab - 8ac
13. 2a(2b + (-4c)) = 4ab - 8ac
14. 3jkb(a + d) = 3abjk +3bdjk
15. 4g(a + b + c +2) = 4ag + 4bg + 4cg + 8g
16. -3(-a -b -c) = 3a + 3b + 3c
17. $-4bc(b + c) = -4b^2c - 4bc^2$
18. $3(k + -3n + \frac{1}{3}j + t^2) = 3k - 9n + j + 3t^2$
19. $3p(k + (-3n) + \frac{1}{3}j + t^2) = 3pk - 9pn + jp + 3pt^2$

Exercise 19

1. $\frac{3p}{2}$ or $1.5p$
2. $\frac{5p}{6}$
3. $\frac{5a}{3}$
4. $\frac{t-k}{7}$
5. $\frac{23a-2}{6}$
6. $\frac{6-b}{2}$
7. -e
8. $\frac{p}{6}$
9. $\frac{3pj-10p}{6}$
10. $\frac{41x}{20}$
11. $\frac{41x+20y}{20}$
12. $\frac{3r+3s+t}{3}$
13. t
14. 3g
15. $\frac{3w}{5}$
16. $\frac{19sw}{12}$
17. 0

Exercise 20

1. $\frac{ab}{2}$
2. ab
3. ab
4. $\frac{6}{ab}$
5. $\frac{ab}{c}$
6. $-\frac{3ak}{2}$
7. $-\frac{c}{5ab}$
8. -av
9. $\frac{abc}{2}$

10. $-\frac{ac}{bk}$
11. $\frac{ab}{e}$
12. -abc
13. abcx
14. $\frac{20a}{8} = \frac{5a}{2}$
15. $\frac{a^2b^2}{2}$

Exercise 21
1. natural: $\{1, 10, 34, 202\}$
   whole: $\{0, 1, 10, 34, 202\}$
   negative integers: $\{-3\}$
   integers: $\{-3, 0, 1, 10, 34, 202\}$
   rational numbers: all except $\{\sqrt{2}, \pi\}$
   irrational numbers: $\{\sqrt{2}, \pi\}$

2. 10 > 9.99 > 1 > 0.5 > 0 > -0.001 > $-\frac{1}{4}$ > $-\frac{1}{2}$ > -3 > -3.01
3. -23
4. 0
5. -12
6. 0
7. -1
8. no. quotient of variables
9. (a) $\frac{1}{2}$, 0.5, -2, 3   (b) 4
10. -1
11. 8a -6b
12. k
13. 5a + 5b
14. 6
15. $\frac{9}{2}x + 1.5y$
16. $\frac{6x+18y-6a}{12}$
17. 3x
18. x-4y
19. -2a - b
20. abcde

Exercise 22
1. a = 3
2. s = 20
3. b = 3
4. c = 9
5. c = 9
6. b = 3
7. x = 22
8. x = 0
9. y = -14
10. k = 54
11. z = -14
12. x (note: This is not an equations, so we just combine terms.
13. m = -27
14. x = 5
15. n = 5
16. t = -14
17. r = 23
18. k = -48
19. s = -68
20. x = 0

Exercise 23
1. x = 4
2. x = -1
3. a = 100
4. s = 100
5. y = -4
6. t = -4
7. b = -27
8. z = -8
9. x = -9
10. u = -27
11. a = -80
12. w = -16
13. y = 12
14. a = 50
15. b = 147
16. y = -10
17. u = 4
18. z = -24
19. r = 6
20. z = 12
21. x = 0
22. x = 1

Exercise 24
1. x = 10
2. x = -4
3. x = 10
4. y = 4
5. x = 30
6. b = 11
7. x = 42
8. n = -81
9. x = 0
10. k = 3
11. x = $\frac{1}{6}$
12. p = 1
13. a = -25
14. a = 20
15. b = 24

16. b = 5
17. c = -12
18. r = -24
19. r = 24
20. g = 33
21. s = -34
22. t = -94/3
23. t = 8/9
24. w = 1/15
25. x = 10
26. x = -4

Final Problems

1. True
2. False
3. True
4. True
5. Read Balance Rule
6. Read Isolation Rule
7. -20
8. +20 or 20
9. a + t
10. a - t
11. at or a· t or a x t
12. $\frac{a}{t}$ or a ÷ t
13. x + at
14. x - $\frac{a}{t}$
15. at + $\frac{a}{t}$
16. $\underline{3}\,x^2 + \underline{0.5}\,y + \underline{\left(-\frac{1}{2}\right)}\,a$
17. >
18. >
19. 6
20. 6
21. 7
22. 1
23. 3
24. 1
25. 0
26. -6
27. -6
28. 9
29. -15q
30. 0
31. -120abcde
32. ac
33. $\frac{100ac+b-2d}{100}$
34. -15 -3a -3b
35. -15 -3a -3b
36. $\frac{7a}{12}$
37. abcde

1. t = -24
2. r = 24
3. r = 26
4. p = -7
5. x = -63
6. x = -3
7. z = -7
8. x = -4
9. w = -63
10. x = 0
11. x = -24
12. x = $\frac{1}{2}$
13. x = 15
14. x = $\frac{125}{21}$
15. x = 3
16. x = $\frac{20}{29}$
17. x = $\frac{105}{52}$